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FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

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Quantum Electrodynamics (QED) with Intense Background Fields

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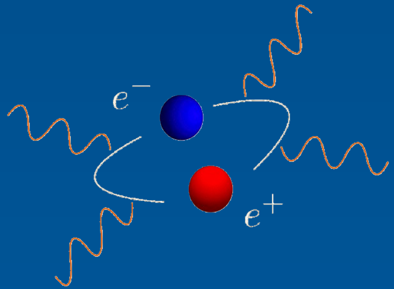


Motivation

[Fedotov, *et al.*: Phys. Rep. **1010** (2023)]

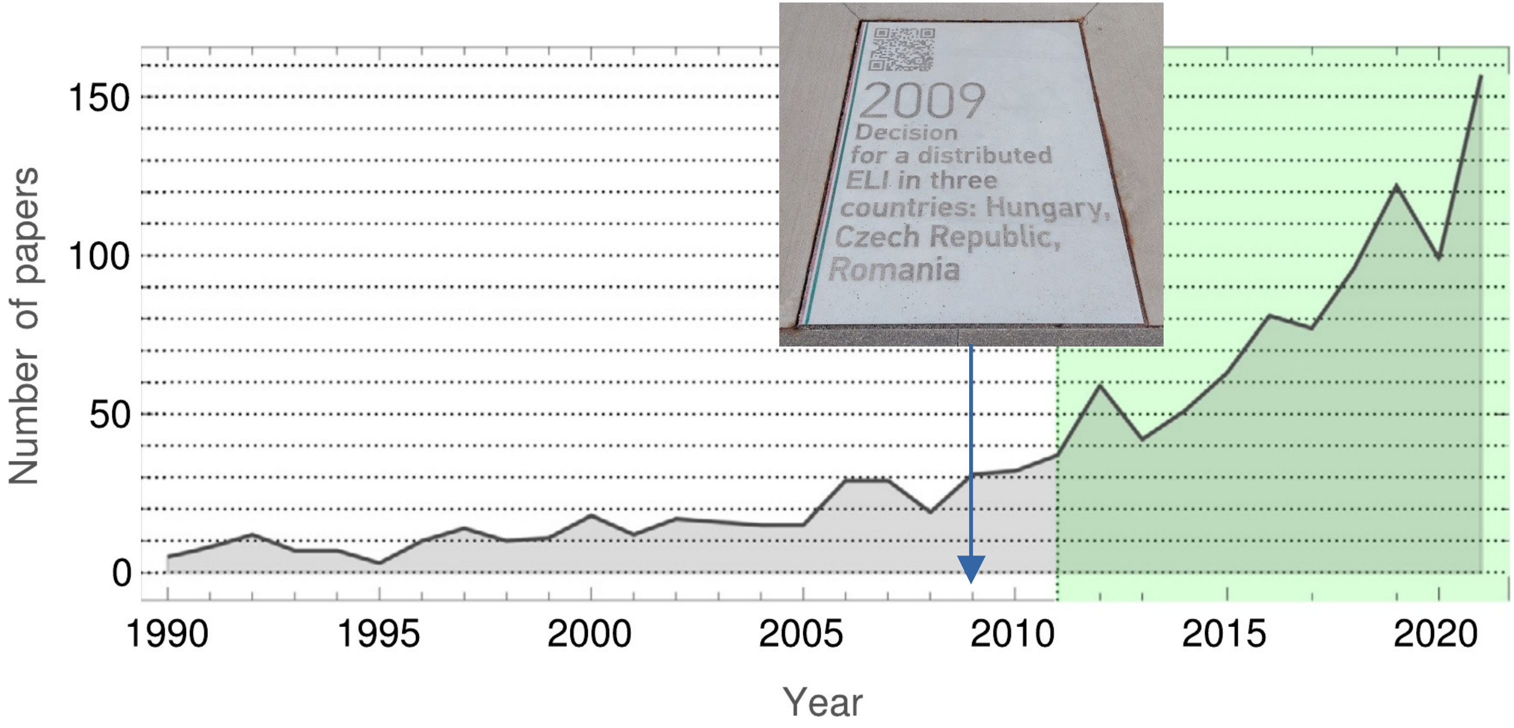
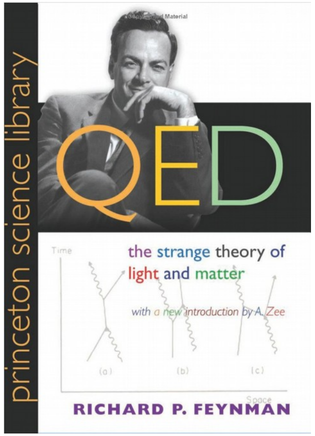


Fig. 1. Indicative bibliometric search using NASA-ADS, for at least one of the following terms occurring in the abstract: "strong field QED", "nonlinear QED", "nonlinear Compton", "nonlinear Breit-Wheeler", "locally constant field", "Schwinger effect", "Schwinger pair".

Contents

1 Introduction

Consider quantum electrodynamics (QED) $(\hbar = c = \epsilon_0 = 1)$



$\rightarrow$ defined by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{QED}} &= \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + e\bar{\psi}\not{a}\psi \\ &= \bar{\psi}(\mathrm{i}\not{\partial} - m)\psi + \frac{1}{2} a^{\mu}(\partial^2 g_{\mu\nu} - \partial_\mu \partial_\nu) a^{\nu} + e\bar{\psi}\not{a}\psi\end{aligned}$$

- quantum fields/particles:

  $\rightarrow$ e^+/e^- field ψ (mass m, charge $|e|$, $\bar{\psi} = \psi^\dagger \gamma^0$) $\leftarrow$ 4-component complex Dirac spinor, anti-commuting/*Grassmann-valued* field

  $\rightarrow$ photon field a^μ (field strength tensor $f^{\mu\nu} = \partial^\mu a^\nu - \partial^\nu a^\mu$) $\leftarrow$ U(1) gauge field

- γ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}\mathbb{1}$ ($\not{C} = C_\mu \gamma^\mu$, such that $\not{C}^2 = -C^2$) $\quad g^{\mu\nu} = \mathrm{diag}(-,+,+,+)$

1 Introduction

- all physical information of a quantum field theory (QFT) stored in correlation functions

$\leftrightarrow$ knowing these $\hat{=}$ knowing all the physics

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle := \langle 0|T\big[\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\big]|0\rangle = \mathcal{N}\int\mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)\,\mathrm{e}^{\mathrm{i}S[\phi]}$$

$\rightarrow$ $S[\phi]=\int \mathrm{d}^4x\,\mathcal{L}$ is the microscopic action of the theory with Lagrangian $\mathcal{L}\xrightarrow{\text{QED}}\mathcal{L}_{\text{QED}}$

$\rightarrow$ ϕ are quantum fields $\xrightarrow{\text{QED}}$ $\phi=(\bar{\psi},\psi,a^\mu)\,,\quad \int\mathcal{D}\phi\rightarrow\int\mathcal{D}a\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi$

$\rightarrow$ $\mathcal{N}$ is a normalization; conventionally $\mathcal{N}^{-1}=\int\mathcal{D}\phi\,\mathrm{e}^{\mathrm{i}S[\phi]}$

- relation to S-matrix elements (scattering amplitudes with given *in* and *out* states)

  via Lehmann-Symanzik-Zimmermann (LSZ) reduction reduction formula

1 Introduction

- many (functional) integrals need to be evaluated $\leftrightarrow$ can do only Gaussian ones

$$\mathcal{N}\int\mathcal{D}\phi\ \mathrm{e}^{\mathrm{i}\int\mathrm{d}^4x\,\frac{1}{2}\phi M\phi} = \det{}^{-\frac{1}{2}}M\,,\quad \mathcal{N}\int\mathcal{D}\bar\psi\int\mathcal{D}\psi\ \mathrm{e}^{\mathrm{i}\int\mathrm{d}^4x\,\bar\psi M\psi} = \det M$$

1 Introduction

- many (functional) integrals need to be evaluated $\leftrightarrow$ can do only Gaussian ones

  $\rightarrow$ resort to perturbative expansion!

- let's see how this works in QED (modulo gauge fixing) $$\int \mathcal{D}\bar\psi \int \mathcal{D}\psi\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \bar\psi M \psi} = \det M$$

$$\int \mathcal{D}\phi\, \mathcal{F}[\phi]\, \mathrm{e}^{\mathrm{i}S[\phi]} \xrightarrow{\text{QED}} \int \mathcal{D}a \int \mathcal{D}\bar\psi \int \mathcal{D}\psi\, \mathcal{F}[\bar\psi,\psi,a^\mu]\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \{\bar\psi(\mathrm{i}\partial\!\!\!/ - m)\psi + \frac{1}{2}a^\mu(\partial^2 g_{\mu\nu} - \partial_\mu\partial_\nu)a^\nu + e\bar\psi a\!\!\!/\psi\}}$$

$$= \int \mathcal{D}a \int \mathcal{D}\bar\psi \int \mathcal{D}\psi\, \mathcal{F}[\bar\psi,\psi,a^\mu] \sum_{n=0}^{\infty} \frac{1}{n!}\Big(\mathrm{i}e \int \mathrm{d}^4x\, \bar\psi a\!\!\!/\psi\Big)^n\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \{\bar\psi(\mathrm{i}\partial\!\!\!/ - m)\psi + \frac{1}{2}a^\mu(\partial^2 g_{\mu\nu} - \partial_\mu\partial_\nu)a^\nu\}}$$

$\leftrightarrow$ at each given order in n $\hat{=}$ Gaussian integrals

- for $n=0$ above expression factorizes in theories of free (non-interacting) Dirac fermions and photons

1 Introduction

- free fermion and photon propagators in position space follow as

$$\langle\bar\psi(y)\psi(x)\rangle\big|_{e=0} = \mathrm{i}G(x,y) \qquad (\mathrm{i}\partial\!\!\!/ - m)G(x,y) = \delta(x-y)\mathbb{1}$$

$$\langle a^\nu(y)a^\mu(x)\rangle\big|_{e=0} = \mathrm{i}D^{\mu\nu}(x,y) \qquad \big(\partial^2 g_{\mu\nu} - \partial_\mu\partial_\nu\big)D^{\mu\nu}(x,y) = \delta(x-y) \qquad \text{(modulo gauge fixing)}$$

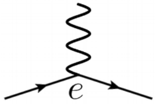
- translational invariance of field-free vacuum $\leftrightarrow$ $G(x,y) = G(x-y)\,,\ D^{\mu\nu}(x,y) = D^{\mu\nu}(x-y)$

  $\rightarrow$ momentum space $G(p)\,,\ D^{\mu\nu}(p)$

- pictorial representation ($\hat{=}$ Feynman rules)

$D^{\mu\nu} =$ 

$G =$ 

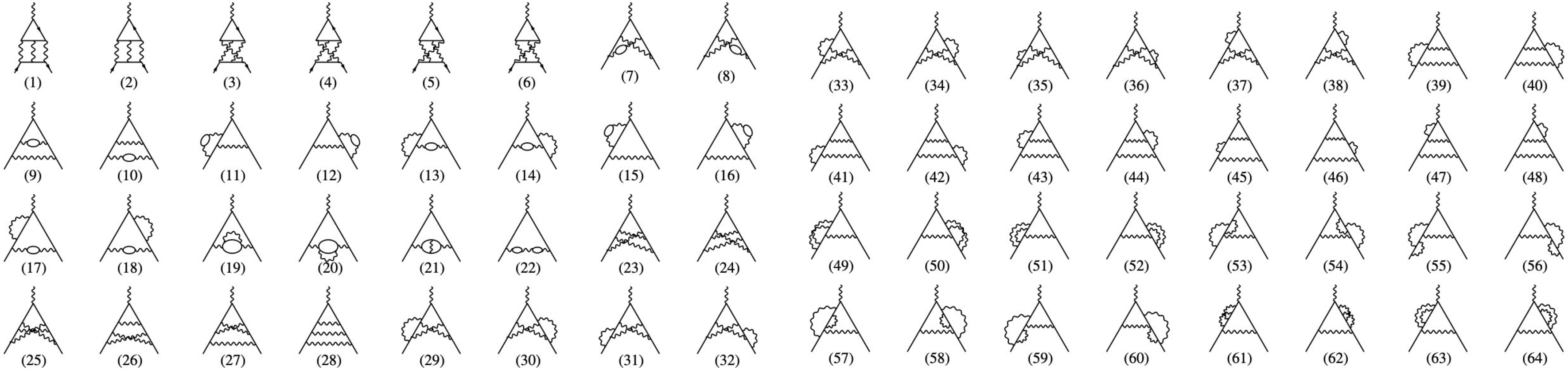
$\mathrm{i}e\gamma^\mu =$ 

- predictions for generic processes possible resorting to expansion in $e \rightarrow \alpha = \frac{e^2}{4\pi}$ on probability level

- for standard QED this works impressively well because of $\alpha(\mu = m) \simeq \frac{1}{137} \ll 1$
- prime example: anomalous magnetic moment of the electron "$g-2$"

$\leftrightarrow$ the g-factor measures the proportionality between magnetic moment $\vec{\mu}$ and spin $\vec{S}$ of the electron

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$ arises from vertex correction



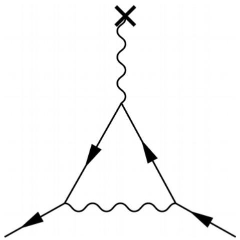
[Jegerlehner, Nyffeler: Phys. Rep. **477** (2009)]

1 Introduction

- for standard QED this works impressively well because of $\alpha(\mu = m) \simeq \frac{1}{137} \ll 1$
- prime example: anomalous magnetic moment of the electron "$g-2$"

$\leftrightarrow$ the g-factor measures the proportionality between magnetic moment $\vec{\mu}$ and spin $\vec{S}$ of the electron

$\vec{\mu} = g \frac{e}{2m} \vec{S}$ arises from vertex correction @ **one** loop:



$\rightarrow$ theory: 5-loop computation (12672 5-loop Feynman diagrams)

$$\frac{g-2}{2}\Big|_{\mathrm{th}} = \frac{1}{2}\frac{\alpha}{\pi} - 0.32848\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.18124\ldots\left(\frac{\alpha}{\pi}\right)^3 - 1.9112\ldots\left(\frac{\alpha}{\pi}\right)^4 + 6.599\ldots\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$+$ independent α-measurement from atomic recoil of Rb

$\rightarrow$ experiment: g measured directly with single-e^- cyclotron, without measuring α

1 Introduction

→ remarkable agreement between theory and experiment

$$\left.\frac{g-2}{2}\right|_{\rm th} = 0.00115\,|\,96\,|\,52\,|\,18\,|\,059(13)$$ [Aoyama, Kinoshita, Nio: Phys. Rev. D **97** (2018)]

$$\left.\frac{g-2}{2}\right|_{\rm exp} = 0.00115\,|\,96\,|\,52\,|\,18\,|\,2031(720)$$ [Fan, Myers, Sukra, Gabrielse: Phys. Rev. Lett. **130** (2023)]

2 3 4 5 th order in α

Apparently everything in QED is well-tested and understood! ← this is not true at all

- many open questions and problems to be resolved

→ very little is known beyond perturbation theory

→ intense background fields provide a means to study this direction ← in theory and experiment

Recent review: [Fedotov, *et al.*: Phys. Rep. **1010** (2023)]

2 Strong field QED

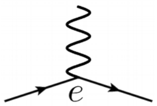
From QED to "strong field QED" (sfQED)

$\rightarrow$ consider the QED Lagrangian

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + e\bar{\psi} a\!\!\!/ \psi$$

$D^{\mu\nu} =$ 

$G =$ 

$\mathrm{i}e\gamma^{\mu} =$ 

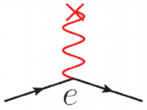
+ external (non-quantized) electromagnetic field

$\leftrightarrow$ gauge potential A^{μ} ; field strength tensor $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$

$\leftrightarrow$ inhibit any direct coupling between quantum photon field a^{μ} and external field A^{μ}

$\rightarrow$ arrive at sfQED Lagrangian

$$\mathcal{L}_{\text{sfQED}} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + e\bar{\psi}(a\!\!\!/ + A\!\!\!/)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

new vertex: $\mathrm{i}eA\!\!\!/ =$ 

2 Strong field QED

- introduce background covariant derivative: $D_\mu[A] = \partial_\mu - \mathrm{i}eA_\mu$
- a trivial rewriting of the Lagrangian yields the alternative representation

$$\mathcal{L}_{\mathrm{sfQED}} = \bar\psi(\mathrm{i}\not{D}[A] - m)\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + e\bar\psi\not{a}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$ "Furry picture"

$\leftrightarrow$ closely resembles standard QED

$$\mathcal{L}_{\mathrm{QED}} = \bar\psi(\mathrm{i}\not{\partial} - m)\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} + e\bar\psi\not{a}\psi$$

$\rightarrow$ replace $\partial_\mu \rightarrow D_\mu[A]$ and add Maxwell term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ of external field

- suggests to exactly account for the external field dependence in the fermion propagator

"to all orders in eA^μ" [Furry: Phys. Rev. **81** (1951)]

2 Strong field QED

- proceed as before for standard QED $\displaystyle\int \mathcal{D}\phi\ \mathcal{F}[\phi]\ \mathrm{e}^{\mathrm{i}S[\phi]}$

$$\xrightarrow{\text{sfQED}} \int \mathcal{D}a \int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi\ \mathcal{F}[\bar{\psi},\psi,a^\mu] \sum_{n=0}^{\infty} \frac{1}{n!} \Big(\mathrm{ie} \int \mathrm{d}^4x\, \bar{\psi} \not{a} \psi\Big)^n \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \{\bar{\psi}(\mathrm{i}\not{D}[A]-m)\psi\ +\ \frac{1}{2}a^\mu(\partial^2 g_{\mu\nu}-\partial_\mu\partial_\nu)a^\nu\}}$$

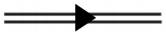
- consider the combination eA^μ as fixed and effectively e - independent $\quad\leftrightarrow\quad$ eA^μ finite even for $e=0$

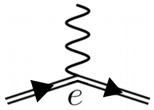
- ~~free~~ fermion and photon propagators in position space follow as

$$\langle\bar{\psi}(y)\psi(x)\rangle\big|_{e=0} = \mathrm{i}G(x,y|A) \qquad \big(\mathrm{i}\not{D}[A]-m\big)G(x,y|A) = \delta(x-y)\mathbb{1}$$

$$\langle a^\nu(y)a^\mu(x)\rangle\big|_{e=0} = \mathrm{i}D^{\mu\nu}(x,y) \qquad \big(\partial^2 g_{\mu\nu}-\partial_\mu\partial_\nu\big)D^{\mu\nu}(x,y) = \delta(x-y) \qquad \text{(modulo gauge fixing)}$$

- Feynman rules: $D^{\mu\nu} =$ 

$G[A] =$ 

$\mathrm{ie}\gamma^\mu =$ 

2 Strong field QED

- in general the external field breaks translational invariance $\rightarrow$ $G(x,y|A)\,,\ D^{\mu\nu}(x-y)$
- all-order in eA^μ studies require solution of $(\mathrm{i}\not{D}[A]-m)G(x,y|A) = \delta(x-y)\mathbb{1}$ for $G(x,y|A)$
- explicit solutions available/possible only for specific fields

$\rightarrow$ Sauter pulse $\mathcal{E}(x) = \mathcal{E}_0/\cosh^2(kx)$ $\leftarrow$ one-dimensional dependence [Sauter: Z. Phys. **53** (1931)]

$\rightarrow$ light-like plane-wave field $A^\mu = \sum_p A_{(p)}(n\cdot x)\,\epsilon^\mu_{(p)}$ [Volkov: Z. Phys. **94** (1935)]

$p\in\{1,2\}$ orthogonal polarizations transverse to n^μ $\leftarrow$ "unit (four) wave vector"

$\leftrightarrow$ polarization vectors $\epsilon^\mu_{(p)}$; 'lightfront time' $n\cdot x\,,\quad n^2=0\,,\quad \epsilon_{(p)}\cdot\epsilon_{(p')}=\delta_{pp'}\,,\quad n\cdot\epsilon_{(p)}=0$

$\rightarrow$ generic constant electromagnetic fields $\partial_\rho F^{\mu\nu}=0$ [Schwinger: Phys. Rev. **82** (1951)]

(constant crossed field for $\partial^\mu A_{(p)}(n\cdot x)\ \rightarrow$ const.)

- in general the external field breaks translational invariance $\rightarrow$ $G(x,y|A)\,,\ D^{\mu\nu}(x-y)$
- all-order in eA^μ studies require solution of $\big(\mathrm{i}\not{D}[A]-m\big)G(x,y|A) = \delta(x-y)\mathbb{1}$ for $G(x,y|A)$
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$$G(x,y|A)=\int\frac{\mathrm{d}^4p}{(2\pi)^4}\left(1-\frac{\not{n}\not{A}(n\cdot x)}{2n\cdot p}\right)\frac{\not{p}-m}{p^2+m^2-\mathrm{i}0^+}\left(1-\frac{\not{A}(n\cdot y)\not{n}}{2n\cdot p}\right)\mathrm{e}^{\mathrm{i}p\cdot(x-y)+\mathrm{i}\int_{n\cdot x}^{n\cdot y}\mathrm{d}\phi\,\frac{2A(\phi)\cdot p-A^2(\phi)}{2n\cdot p}}$$

2 Strong field QED

Plane-wave field $\hat{=}$ reasonable model of a (pulsed) **laser field**

$\leftrightarrow$ no non-trivial spatial dependence $\leftarrow$ infinitely extended in transverse directions, no focusing, ...

When do we need to account for "all orders in eA^μ"? $\rightarrow$ let's survey the relevant parameters!

$\rightarrow$ "laser field" $A^\mu = \sum_p A_{(p)}(n\cdot x)\,\epsilon^\mu_{(p)} \sim \frac{\mathcal{E}}{\omega}$ $\rightarrow$ $eA^\mu \sim \frac{e\mathcal{E}}{\omega}$ $\leftrightarrow$ $[eA^\mu] = \text{mass}$

$\omega \hat{=}$ typical frequency scale of variation of field $\leftrightarrow$ dimensionful (reference) scale of QED $\hat{=}$ m

- quantify field strength/intensity by dimensionless and gauge-invariant

classical nonlinearity parameter $\boldsymbol{\xi} = \boldsymbol{a_0} = \frac{e\mathcal{E}}{\omega m} = \frac{e\mathcal{E}\lambda\!\!\!^{-}_C}{\hbar\omega} = 3.8\times10^{-13}\,\frac{\mathcal{E}[\mathrm{V/m}]}{\omega[\mathrm{eV}]}$

$\hat{=}$ energy gained by e^- in field over $\lambda\!\!\!^{-}_C = \frac{\hbar}{cm} \simeq 3.8\times10^{-7}\,\mathrm{m}$ in units of background photon energy

$\boldsymbol{\xi^2} \approx$ measure of number of laser photons interacting with e^-

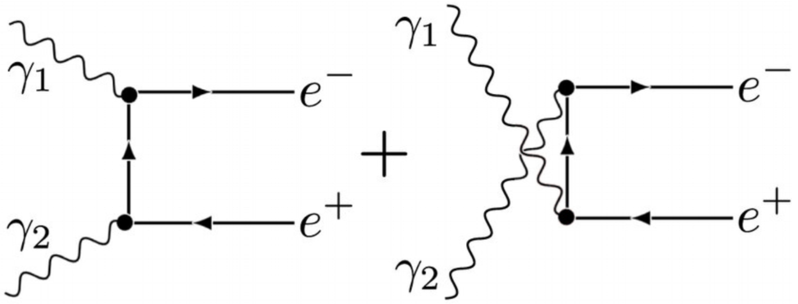
- quantify <u>collision energy</u> of probe particle and plane-wave background by $\boldsymbol{\eta} = -\frac{k\cdot p}{m^2} = -\frac{\omega}{m}\frac{n\cdot p}{m}$

$k^\mu = \omega n^\mu \;\hat{=}\;$ representative wave vector of background field ; $\quad p^\mu \;\hat{=}\;$ probe particle momentum

$\rightarrow$ probe charge: $\boldsymbol{\eta} \;\hat{=}\;$ laser photon energy in rest frame of probe charge in units of m

$\rightarrow$ probe photon: $\boldsymbol{\eta} \;\hat{=}\;$ half center-of-mass-energy-squared in collision with single laser photon $/m^2$

$\leftrightarrow$ linear Breit-Wheeler threshold: $\eta \geq 2$ [Breit, Wheeler: Phys. Rev. **46** (1934)]



DECEMBER 15, 1934 PHYSICAL REVIEW VOLUME 46

Collision of Two Light Quanta

G. Breit* and John A. Wheeler,** *Department of Physics, New York University*

(Received October 23, 1934)

The recombination of free electrons and free positrons and its connection with the Compton effect have been treated by Dirac before the experimental discovery of the positron. In the present note are given analogous calculations for the production of positron electron pairs as a result of the collision of two light quanta. The angular distribution of the ejected pairs is calculated for different polarizations, and formulas are given for the angular distribution of photons due to recombination. The results are applied to the collision of high energy photons of cosmic radiation with the temperature radiation of interstellar space. The effect on the absorption of such quanta is found to be negligibly small.

2 Strong field QED

- quantify collision energy of probe particle and plane-wave background by $\boldsymbol{\eta} = -\frac{k\cdot p}{m^2} = -\frac{\omega}{m}\frac{n\cdot p}{m}$

  $k^\mu = \omega n^\mu \hat{=}$ representative wave vector of background field ; $p^\mu \hat{=}$ probe particle momentum

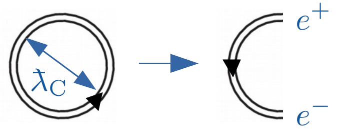
  $\rightarrow$ probe charge: $\boldsymbol{\eta} \hat{=}$ laser photon energy in rest frame of probe charge in units of m

  $\rightarrow$ probe photon: $\boldsymbol{\eta} \hat{=}$ half center-of-mass-energy-squared in collision with single laser photon $/m^2$

- 'quantum nonlinear parameter' $\boldsymbol{\chi} = \frac{e\sqrt{(p\cdot F)^2}}{m^3} \xrightarrow{\text{plane-wave}} -\frac{e\mathcal{E}}{m^2}\frac{n\cdot p}{m} = \boldsymbol{\xi}\boldsymbol{\eta}$

  $\hat{=}$ ratio $\mathcal{E}/E_\mathrm{S}$ in rest frame of probe charge; $E_\mathrm{S} = \frac{c^3}{\hbar}\frac{m^2}{e} \simeq 1.3\times 10^{18}\,\frac{\mathrm{V}}{\mathrm{m}}$ [F. Sauter, Z. Phys. **69** (1931)]

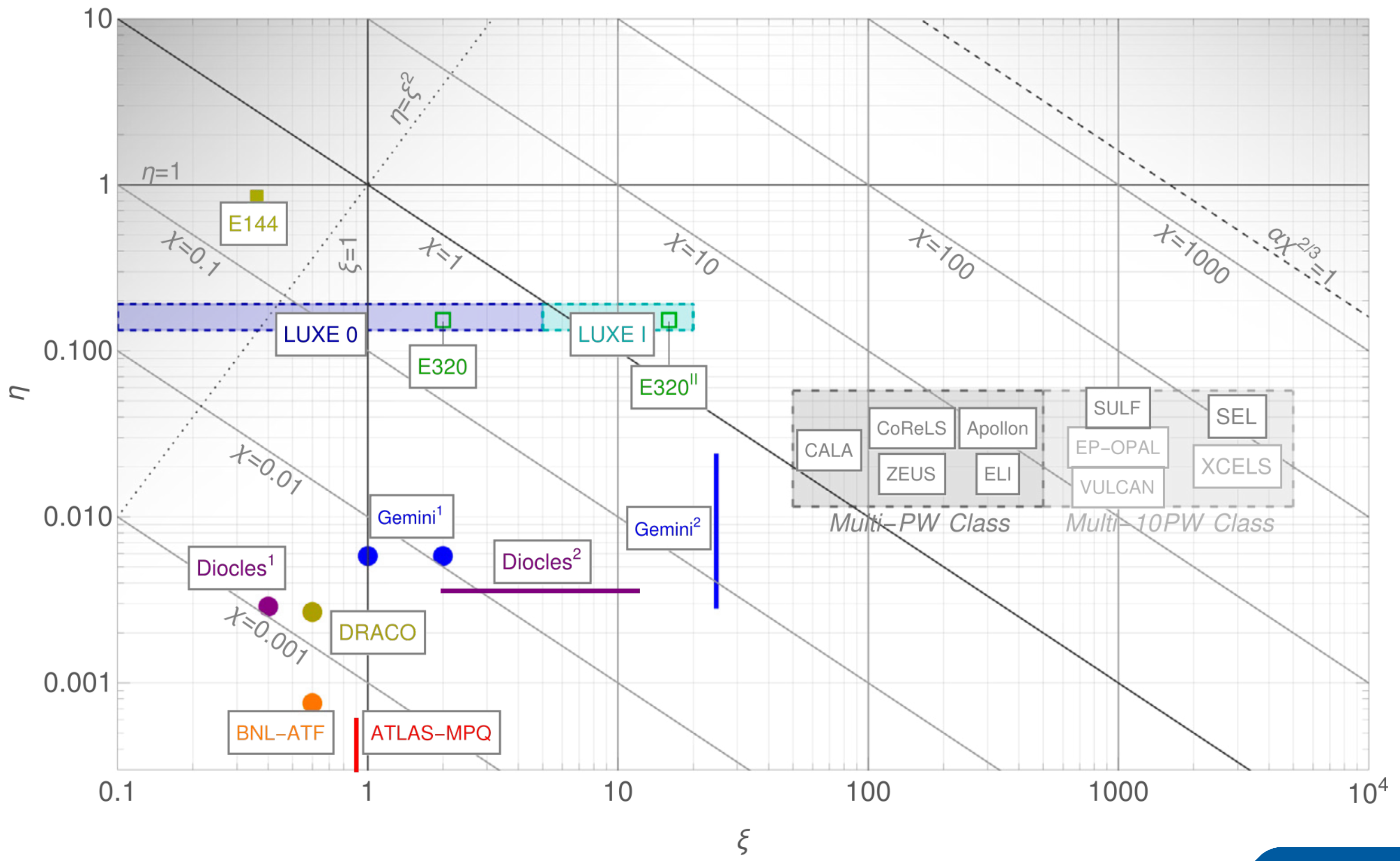
  $\leftrightarrow$ in field of strength E_S : energy gained by e^-/e^+ over $\lambda\!\!\!^{-}_\mathrm{C}$ equals m



$\leftrightarrow$ When do we need to account for "all orders in eA^μ" ? $\rightarrow$ especially for $\chi \gtrsim 1$!

2 Strong field QED

[Fedotov, *et al.*: Phys. Rep. **1010** (2023)]

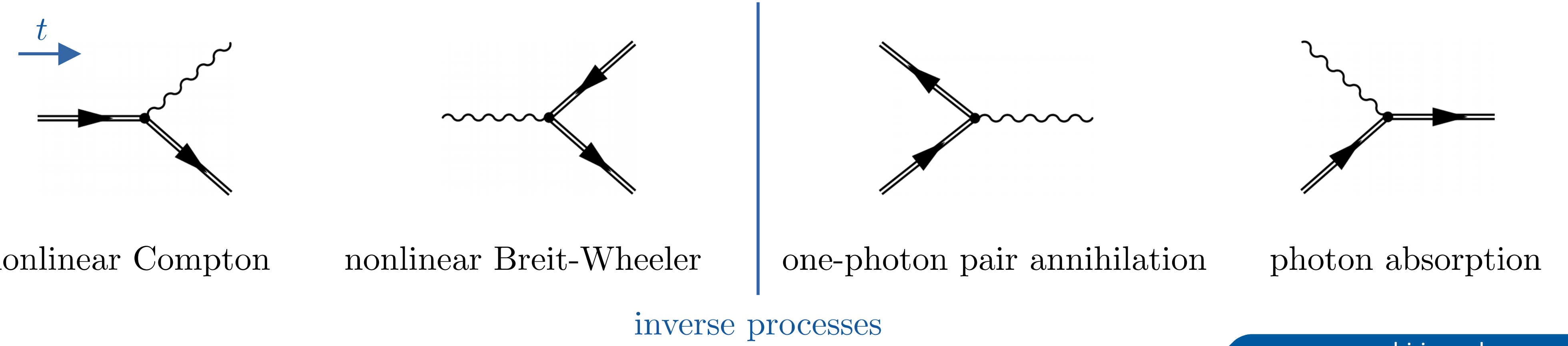


- **beyond single plane wave** also scalar gauge and Lorentz invariants of background relevant

$$\leftrightarrow \quad \mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\vec{B}^2 - \vec{E}^2) \quad \text{and} \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}{}^{\star}F^{\mu\nu} = -\vec{B}\cdot\vec{E}$$

$\rightarrow$ relevant dimensionless ratios: $\frac{e^2\mathcal{F}}{m^4} \sim \frac{e^2\mathcal{G}}{m^4} \sim \left(\frac{\mathcal{E}}{E_{\rm S}}\right)^2$ $\leftarrow$ quantum vacuum effects

1st-order processes possible in plane-wave field: $\hat{=}$ on-shell quantum particles + single (dressed) vertex



3 Quantum vacuum effects

Also Feynman diagrams without in- and outgoing quantum fields contain interesting physics

$\rightarrow$ for a generic QFT these are encoded in

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle \;:=\; \langle 0|T\big[\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\big]|0\rangle \;=\; \mathcal{N}\int\mathcal{D}\phi\;\phi(x_1)\cdots\phi(x_n)\;\mathrm{e}^{\mathrm{i}S[\phi]}$$

with $\phi(x_1)\cdots\phi(x_n)\rightarrow 1$

$\rightarrow$ use QED vacuum as reference $\leftrightarrow$ choose $\mathcal{N}^{-1} = \int\mathcal{D}a\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi\;\mathrm{e}^{\mathrm{i}\int\mathrm{d}^4\mathcal{L}_{\mathrm{QED}}}$

$$\langle 0;\mathrm{out}|0;\mathrm{in}\rangle_A = \frac{\int\mathcal{D}a\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi\;\mathrm{e}^{\mathrm{i}\int\mathrm{d}^4\mathcal{L}_{\mathrm{sfQED}}}}{\int\mathcal{D}a\int\mathcal{D}\bar{\psi}\int\mathcal{D}\psi\;\mathrm{e}^{\mathrm{i}\int\mathrm{d}^4\mathcal{L}_{\mathrm{QED}}}} = \Big\langle \mathrm{e}^{\mathrm{i}\int\mathrm{d}^4x\,\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+e\bar{\psi}\not{A}\psi\}}\Big\rangle_{\mathrm{QED}} =: \mathrm{e}^{\mathrm{i}\Gamma_{\mathrm{HE}}[A]}$$

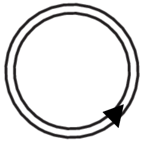
$\leftrightarrow$ Heisenberg-Euler effective action $\Gamma_{\mathrm{HE}}[A]$ [Heisenberg, Euler: Z. Phys. **98** (1936)]

- at one loop $\left(\int \mathcal{D}\bar{\psi} \int \mathcal{D}\psi \; \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\, \bar{\psi} M \psi} = \det M\right)$ [Heisenberg, Euler: Z. Phys. **98** (1936)]

$$\langle 0;\mathrm{out}|0;\mathrm{in}\rangle_A = \mathrm{e}^{\mathrm{i}\Gamma_{\mathrm{HE}}[A]} = \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\,(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})} \frac{\det\big(\mathrm{i}\not{D}[A]-m\big)}{\det\big(\mathrm{i}\not{\partial}-m\big)} + \mathcal{O}(\alpha)$$

$$= \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x\,(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})} \frac{\det\big(G^{-1}[A]\big)}{\det\big(G^{-1}[0]\big)} + \mathcal{O}(\alpha)$$

$$\leftrightarrow \quad \Gamma_{\mathrm{HE}}[A] = \int \mathrm{d}^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) + \mathrm{i}\ln\left(\frac{\det\big(G[A]\big)}{\det\big(G[0]\big)}\right) + \mathcal{O}(\alpha) = \int \mathrm{d}^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) +$$



$\rightarrow$ can be evaluated in cases where $G[A]$ is explicitly known $\quad + \mathrm{O}(\alpha)$

$\leftrightarrow$ explicit solutions available only for specific fields: Sauter pulse, plane-wave, constant field

$\rightarrow$ note that $\left|\langle 0;\mathrm{out}|0;\mathrm{in}\rangle_A\right|^2 = \mathrm{e}^{-2\mathrm{Im}\{\Gamma_{\mathrm{HE}}[A]\}}$ is the probability of the vacuum to persist in A^μ

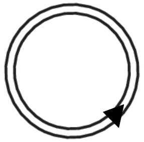
→ Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}. = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} +$$



$$\left(\begin{array}{l}\mathfrak{E},\mathfrak{B}\quad \text{Kraft auf das Elektron.}\\ |\mathfrak{E}_k| = \frac{m^2c^3}{e\hbar} = \frac{1}{\text{„}137\text{“}}\frac{e}{(e^2/mc^2)^2} = \text{„Kritische Feldstärke“.}\end{array}\right)$$

Ihre Entwicklungsglieder für (gegen $|\mathfrak{E}_k|$) kleine Felder beschreiben Prozesse der Streuung von Licht an Licht, deren einfachstes bereits aus einer Störungsrechnung bekannt ist. Für große Felder sind die hier abgeleiteten Feldgleichungen von den Maxwellschen sehr verschieden. Sie werden mit den von Born vorgeschlagenen verglichen.

$$\vec{E}^2 - \vec{B}^2 = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}\,,$$

$$\vec{E}\cdot\vec{B} = -\tfrac{1}{4}F_{\mu\nu}{}^{\star}F^{\mu\nu}$$

The Nobel Prize in Physics 1932
Werner Heisenberg

Werner Heisenberg - Facts

Werner Karl Heisenberg

Born: 5 December 1901, Würzburg, Germany

Died: 1 February 1976, Munich, West Germany (now Germany)

Affiliation at the time of the award: Leipzig University, Leipzig, Germany

Prize motivation: "for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"

Field: quantum mechanics

Werner Heisenberg received his Nobel Prize one year later, in 1933.

Kriegsschicksale Phys. Bl. 45 (1989) Nr. 9

Hans Euler (1909–1941)

Von D. Hoffmann, Berlin*)

Hans Euler, um 1935 (Foto: H. Wergeland, Norwegen/Hurdal)

[Euler, Kockel: Naturwiss. **23** (1935)],
[Euler: Ann. Phys. **418** (1936)]

$\rightarrow$ weak field expansion

$$\mathcal{L}_{\text{HE}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^4}{5760\pi^2 m^4}\left[4\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + 7\left(F_{\mu\nu}{}^{\star}F^{\mu\nu}\right)^2\right] + \ldots$$

$\hat{=}$ "true theory" of electromagnetic fields in vacuum

$\rightarrow$ equations of motion (EOM) $\quad \mathcal{L}(A_\nu, F_{\mu\nu}) \xrightarrow{\text{EOM}} \frac{\partial\mathcal{L}}{\partial A_\nu} - 2\partial_\mu \frac{\partial\mathcal{L}}{\partial F_{\mu\nu}} = 0$

$$\partial_\mu\left(F^{\mu\nu} - \frac{e^4}{720\pi^2 m^4}\left[4\left(F_{\alpha\beta}F^{\alpha\beta}\right)F^{\mu\nu} + 7\left(F_{\alpha\beta}{}^{\star}F^{\alpha\beta}\right){}^{\star}F^{\mu\nu}\right] + \ldots\right) = 0$$

$$\leftrightarrow \quad \partial_\mu\left(F^{\mu\nu} - \frac{4\alpha}{45\pi}\frac{1}{2}\frac{\vec{B}^2 - \vec{E}^2}{E_{\rm S}^2}F^{\mu\nu} + \frac{7\alpha}{45\pi}\frac{\vec{E}\cdot\vec{B}}{E_{\rm S}^2}{}^{\star}F^{\mu\nu} + \ldots\right) = 0\,, \quad \alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

$\rightarrow$ light-by-light scattering phenomena, vacuum birefringence, ...

3 Quantum vacuum effects

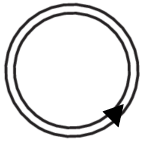
[Schwinger: Phys. Rev. **82** (1951)],
[Nikishov: Sov. Phys. JETP **30** (1970)]

$\rightarrow$ imaginary part $\hat{=}$ e^-/e^+ pair production

$$\mathrm{Im}\{\mathcal{L}_{\mathrm{HE}}\} = \frac{(e\mathfrak{E})^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\,\mathrm{e}^{-n\pi\frac{m^2}{e\mathfrak{E}}}\,\frac{\frac{\mathfrak{B}}{\mathfrak{E}}n\pi}{\tanh\left(\frac{\mathfrak{B}}{\mathfrak{E}}n\pi\right)} + \mathcal{O}(\alpha)$$

$\leftarrow$ manifestly non-perturbative; no Maclaurin series in $\frac{e\mathfrak{E}}{m^2} \ll 1$

with $\mathfrak{E} = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}-\mathcal{F}}$ and $\mathfrak{B} = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}+\mathcal{F}}$

$\leftrightarrow$ diagrammatically: $2\,\mathrm{Im}$  $= \int_{\text{phase space}} \Big|$  $\Big|^2$ $\leftarrow$ optical theorem

$$\leftrightarrow \quad \big|\langle 0;\mathrm{out}|0;\mathrm{in}\rangle\big|^2 = \mathrm{e}^{-2\mathrm{Im}\{\Gamma_{\mathrm{HE}}[A]\}} \simeq \mathrm{e}^{-\int \mathrm{d}^4x\, w(\mathcal{F},\mathcal{G})}$$

$\leftarrow$ local decay rate $w(\mathcal{F},\mathcal{G})$

$$\rightarrow \text{ weak field limit: } \quad w(\mathcal{F},\mathcal{G}) \simeq \frac{(e\mathfrak{E})^2}{4\pi^3}\,\mathrm{e}^{-\pi\frac{m^2}{e\mathfrak{E}}}\,\frac{\frac{\mathfrak{B}}{\mathfrak{E}}\pi}{\tanh\left(\frac{\mathfrak{B}}{\mathfrak{E}}\pi\right)} \xrightarrow[\mathfrak{E}\to E,\ \mathfrak{B}\to 0]{\text{purely electric field}} \frac{(eE)^2}{4\pi^3}\,\mathrm{e}^{-\pi\frac{m^2}{eE}}$$

4 Conclusions + Outlook

In this talk, I have

- introduced you to sfQED, highlighting parallels and differences from ordinary QED,
- made you familiar with the relevant concepts and characteristic parameters,
- argued that sfQED is needed for describing quantum processes in high-intensity laser fields,
- (thereby hopefully) convinced you that sfQED is an interesting and topical research area.

Certainly, much more could be said ...

→ for further details, consider reviews + references therein

[Gonoskov, *et al.*: Rev. Mod. Phys. **94** (2022)]
[Fedotov, *et al.*: Phys. Rep. **1010** (2023)]

There are many interesting open problems to be studied (for instance by you!)

DFG Deutsche Forschungsgemeinschaft
Research Unit FOR 2783

HI JENA
Helmholtz Institute Jena

Thank you very much for the invitation and for your attention!

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