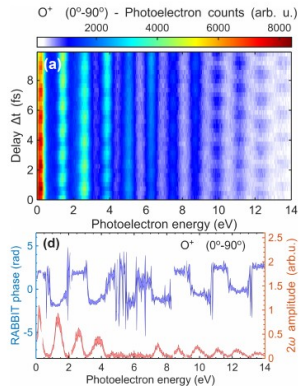


# Attosecond coupling delay in ionization to entangled states of $\text{CO}_2^+$

Ioannis Makos, David Busto, Jakub Benda, Dominik Ertel, Barbara Merzuk, Benjamin Steiner, Fabio Frassetto, Luca Poletto, Claus Dieter Schröter, Thomas Pfeifer, Robert Moshhammer, Serguei Patchkovskii, Van-Hung Hoang, Uwe Thumm, Zdeněk Mašín & Giuseppe Sansone

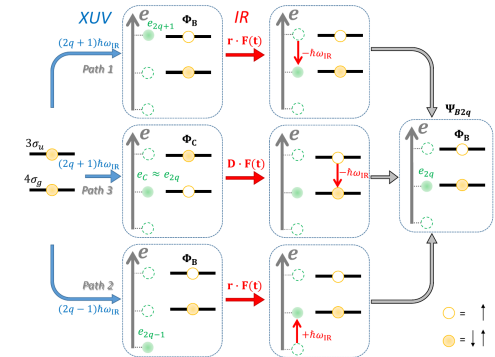


DFG Deutsche Forschungsgemeinschaft

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NEXT NON-LINEAR EXTREME ULTRAVIOLET TO HARDC-RAY TECHNIQUES

GAČR CZECH SCIENCE FOUNDATION



WavemiX 2026, June 2<sup>nd</sup>

## Asymptotic RABBITT delays

RABBITT – measurement of XUV ionization delays  
using  $XUV \pm IR$  two-photon ionization

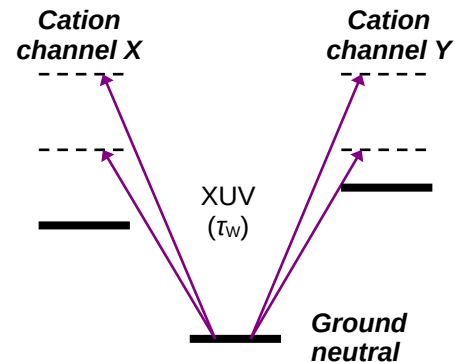
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High-energy separability of RABBITT phases (delays)

$$\tau_R \approx \tau_W$$

$\tau_W$  – photoionization (Wigner) delay



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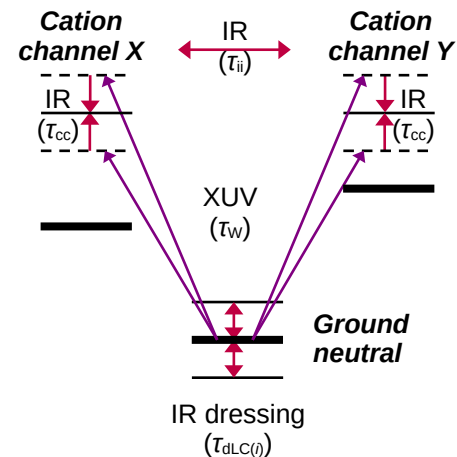
$$\tau_R \approx \tau_W + \tau_{cc} + \tau_{ii} + \tau_{dLC} + \dots$$

$\tau_W$  – photoionization (Wigner) delay

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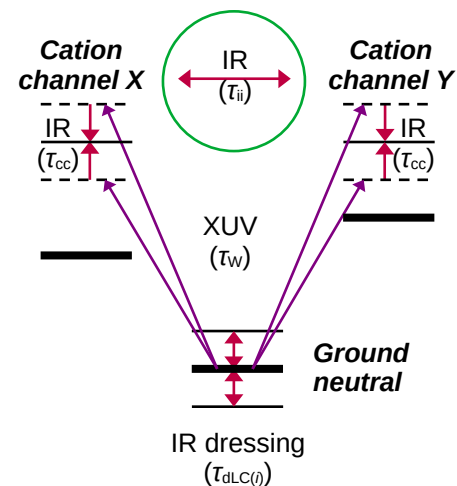
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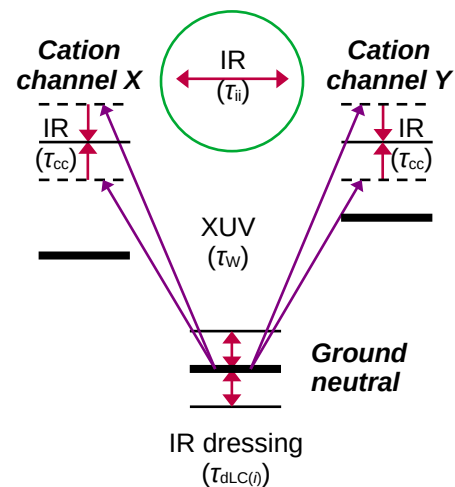
$\tau_{ii}$  – ion-ion coupling delay

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$$2\omega_{IR}\tau_R = \arg M_{XUV-IR} - \arg M_{XUV+IR}$$

$$M = \langle \Psi_f | D | \Psi_{i+\Omega} \rangle$$

$\uparrow$  final continuum state after  $XUV \pm IR$  ionization  
 $\uparrow$  dipole transition operator  
 $\uparrow$  intermediate continuum state after XUV ionization



# Asymptotic RABBITT delays

RABBITT – measurement of XUV ionization delays using XUV±IR two-photon ionization

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$\tau_{cc}$  – continuum-continuum delay

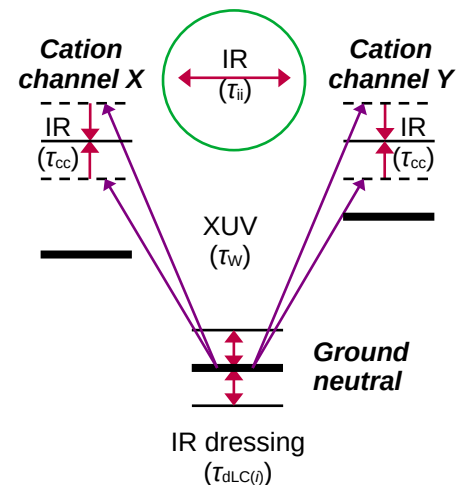
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$\uparrow$  final continuum state after XUV±IR ionization  
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$$D = \mathbf{r}_{pe} \cdot \boldsymbol{\epsilon}_{IR} + \mathbf{D}_{ion} \cdot \boldsymbol{\epsilon}_{IR}$$

$$M = M_{pe} + \delta M_{ion}$$

$$\tau_R = \tau_0 + \delta \tau_{ii}$$

# Asymptotic RABBITT delays

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↑ final continuum state after XUV±IR ionization  
 ↑ dipole transition operator  
 ↑ intermediate continuum state after XUV ionization

$$D = \mathbf{r}_{pe} \cdot \boldsymbol{\epsilon}_I$$

$$M = M_{pe} + \mathcal{O}(M)_{ion}$$

$$\tau_R = \tau_0 + \delta\tau_{ii}$$

PHYSICAL REVIEW A **105**, 053101 (2022)

## Analysis of RABBITT time delays using the stationary multiphoton molecular R-matrix approach

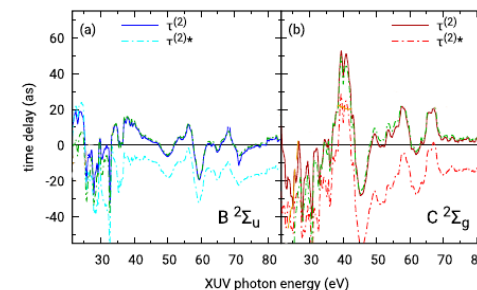
J. Benda<sup>\*</sup> and Z. Mašín<sup>\*</sup>

*Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, Prague 8, 180 00 Czech Republic*

J. D. Gorfinkiel<sup>\*</sup>

*School of Physical Sciences, The Open University, Walton Hall, MK7 6AA Milton Keynes, United Kingdom*

### RABBITT in CO<sub>2</sub> (ion states B & C)



$$M = M_{pe} + \mathcal{O}(M)_{ion}$$

$$\tau_R = \tau_0 + \delta\tau_{ii}$$

# Asymptotic RABBITT delays

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$\uparrow$  final continuum state after XUV±IR ionization  
 $\uparrow$  dipole transition operator  
 $\uparrow$  intermediate continuum state after XUV ionization

PHYSICAL REVIEW A **105**, 053101 (2022)

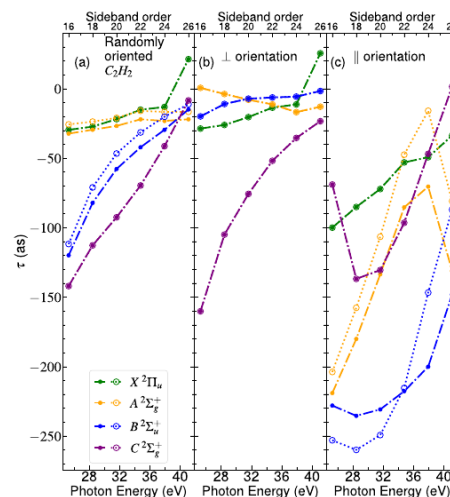
Analysis of RABBITT time delays using the stationary multiphoton molecular R-matrix approach

PHYSICAL REVIEW A **111**, 063107 (2025)

Editors' Suggestion

Three-path interferences in the reconstruction of attosecond beatings by interference of two-photon transitions in molecules

Jorge Delgado<sup>1,2</sup>, Celso M. González-Collado<sup>2</sup>, Piero Decleva<sup>3</sup>, Alicia Palacios<sup>2,4,5,\*</sup> and Fernando Martín<sup>1,2,†</sup>



RABBITT in C<sub>2</sub>H<sub>2</sub>

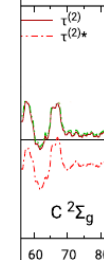
Dipole-coupled states: A & B

Full vs. empty circles: theory with/without A-B dipole coupling.

es, Charles University, public

Milton Keynes, United Kingdom

B & C)



# Asymptotic RABBITT delays

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High-energy separability of RABBITT phases

$$\tau_R \approx \tau_W + \tau_{CC} + \tau_{ii} + \tau_{dLC} + \dots$$

$\tau_W$  – photoionization (Wigner) delay

$\tau_{CC}$  – continuum-continuum delay

$\tau_{ii}$  – ion-ion coupling delay

$\tau_{dLC}$  – dipole-laser coupling delay

Experimentally difficult to access

- states separated by IR quantum
- MB of one state overlaps SB of other

→ coincidence measurement needed

PHYSICAL REVIEW A **105**, 053101 (2022)

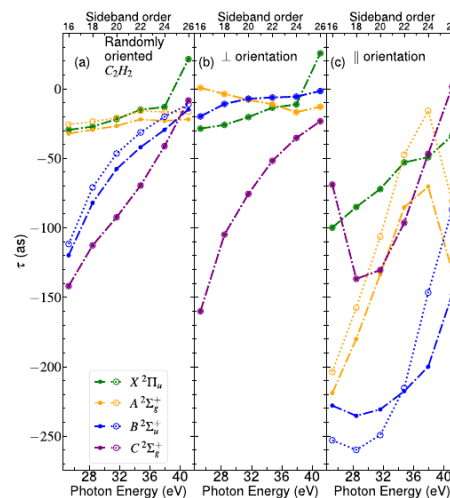
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RABBITT in  $C_2H_2$

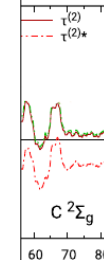
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Full vs. empty circles: theory with/without A-B dipole coupling.

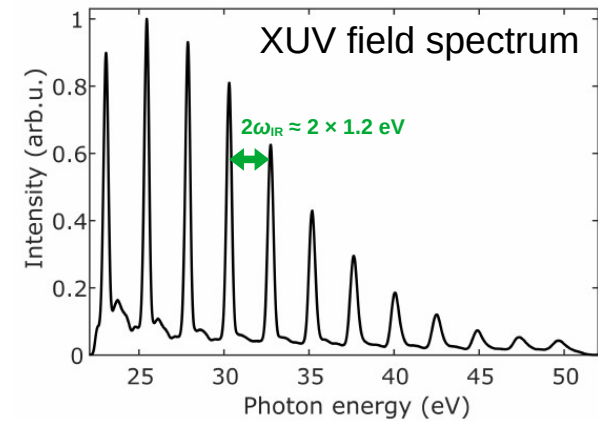
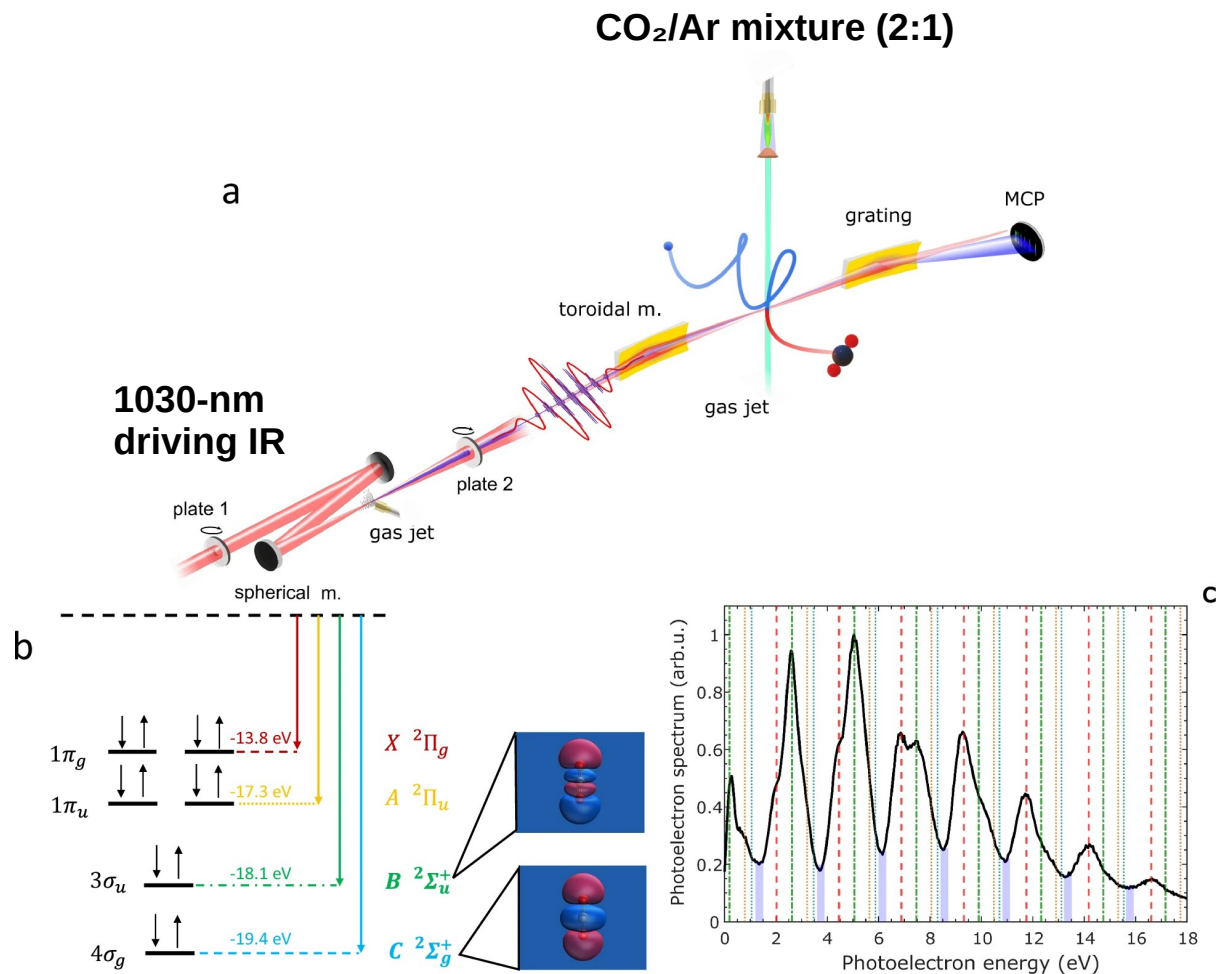
es, Charles University, public

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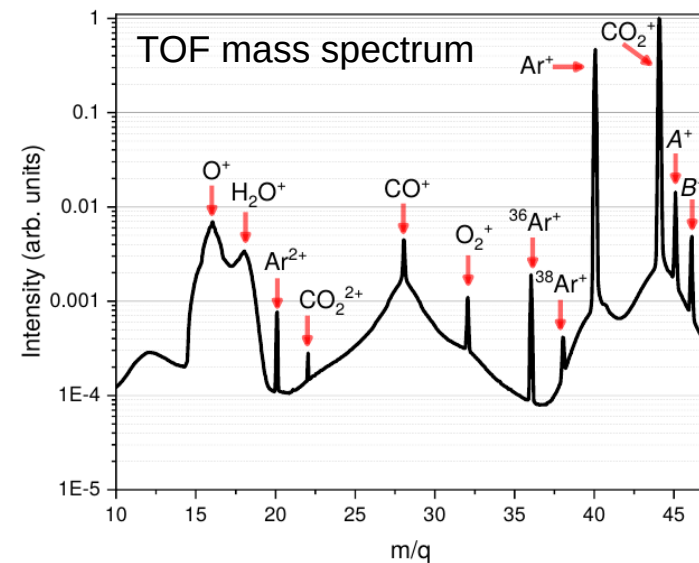
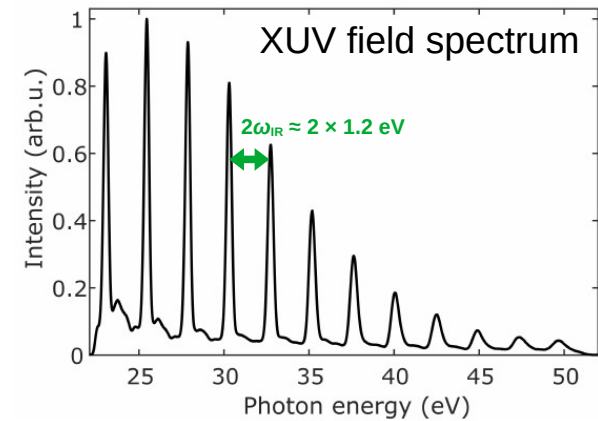
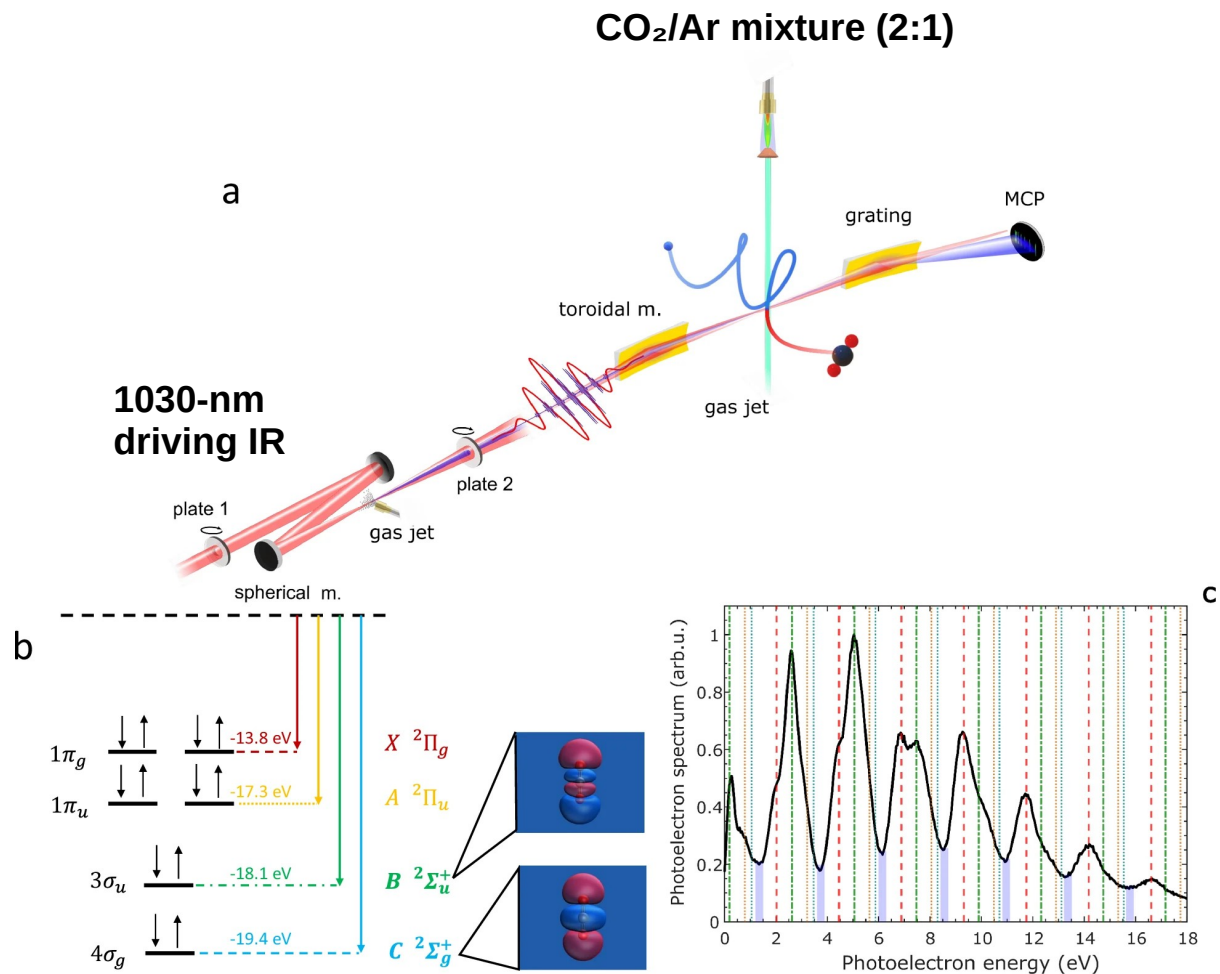
B & C)



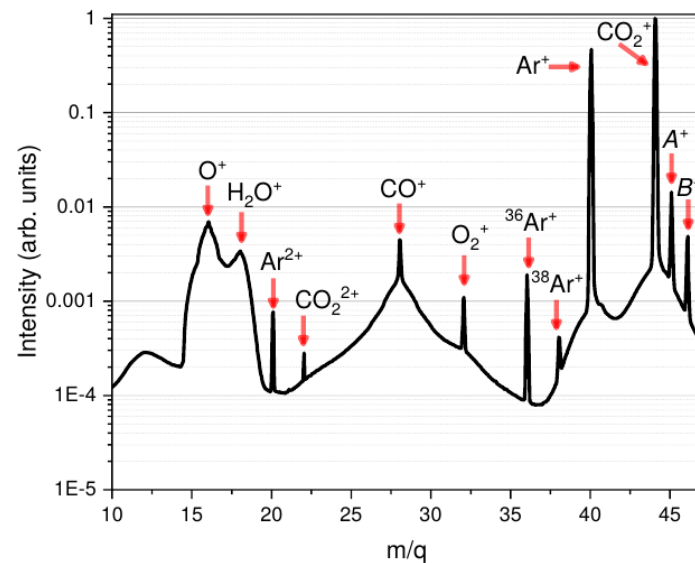
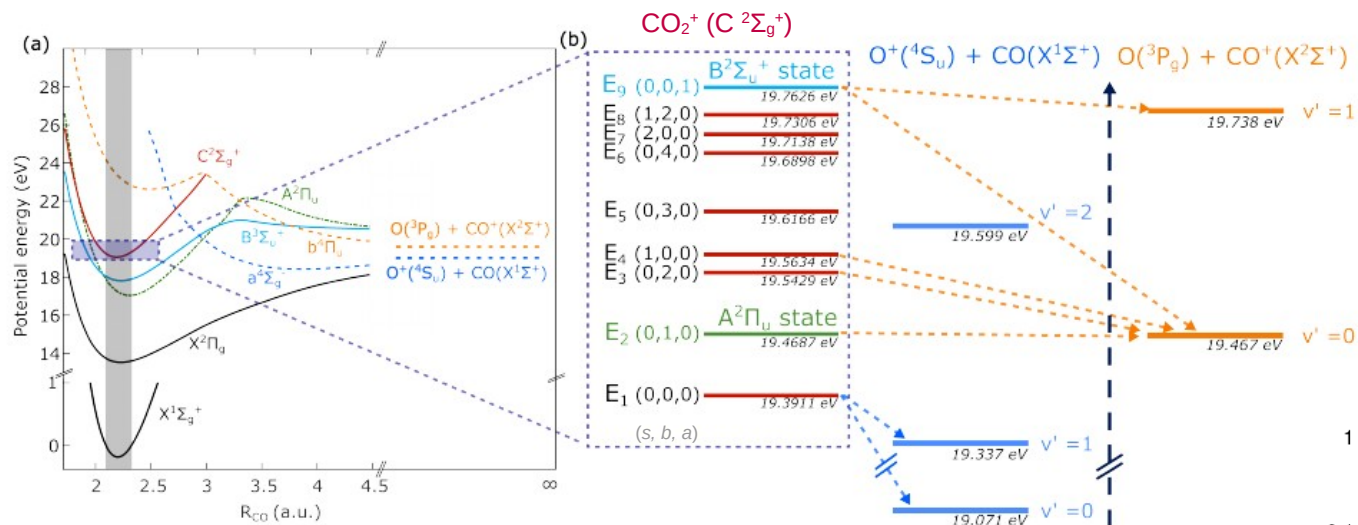
# Experimental method



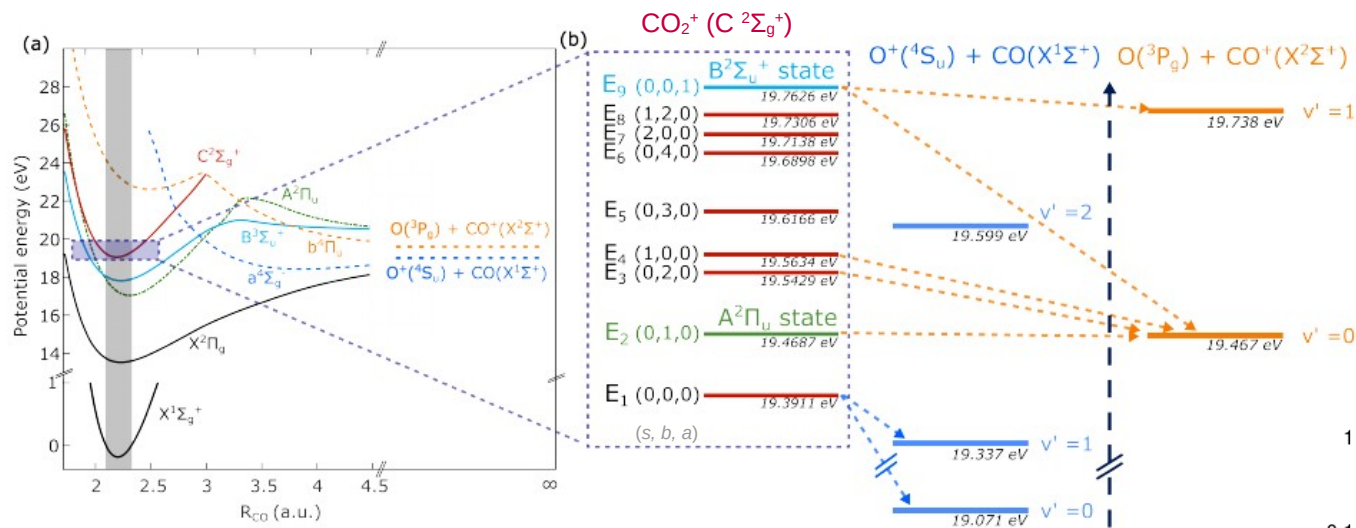
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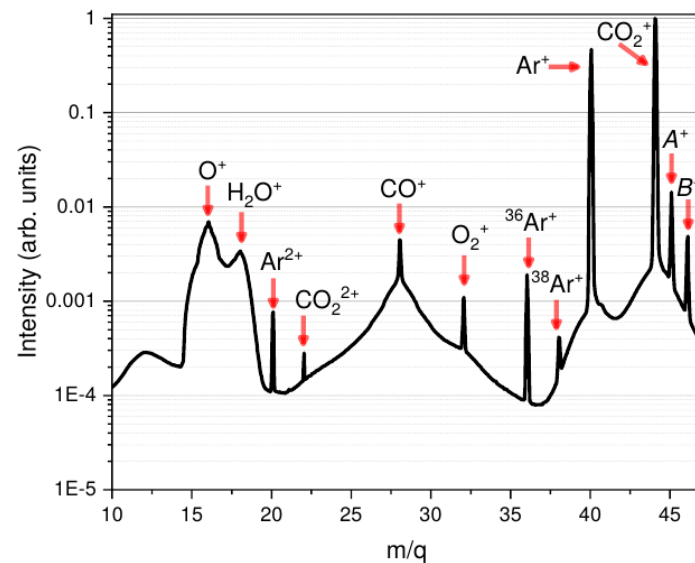
# CO<sub>2</sub><sup>+</sup> potential surfaces



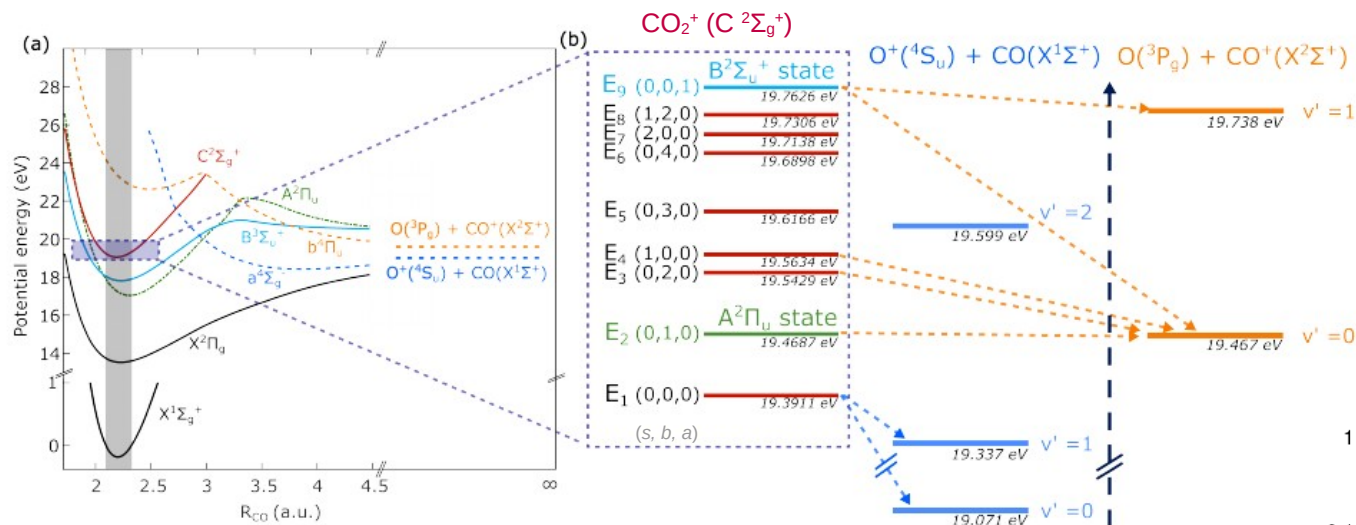
# CO<sub>2</sub><sup>+</sup> potential surfaces



CO<sub>2</sub><sup>+</sup> B  $^2\Sigma_u^+$  ( $v=0$ ) is stable



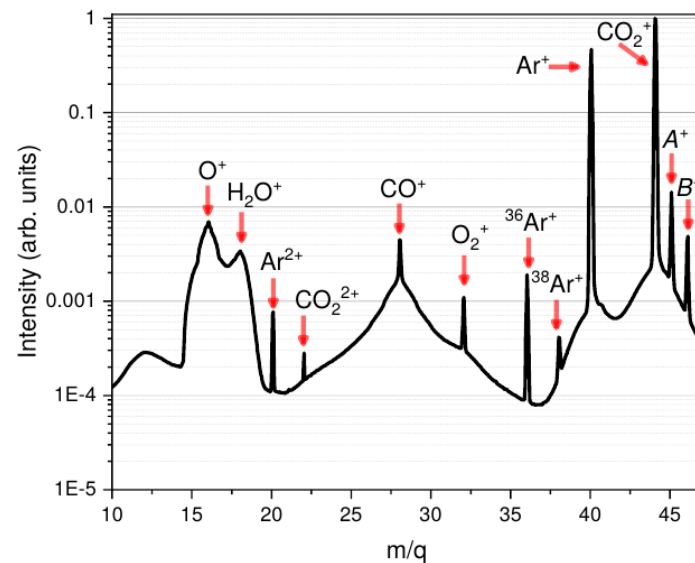
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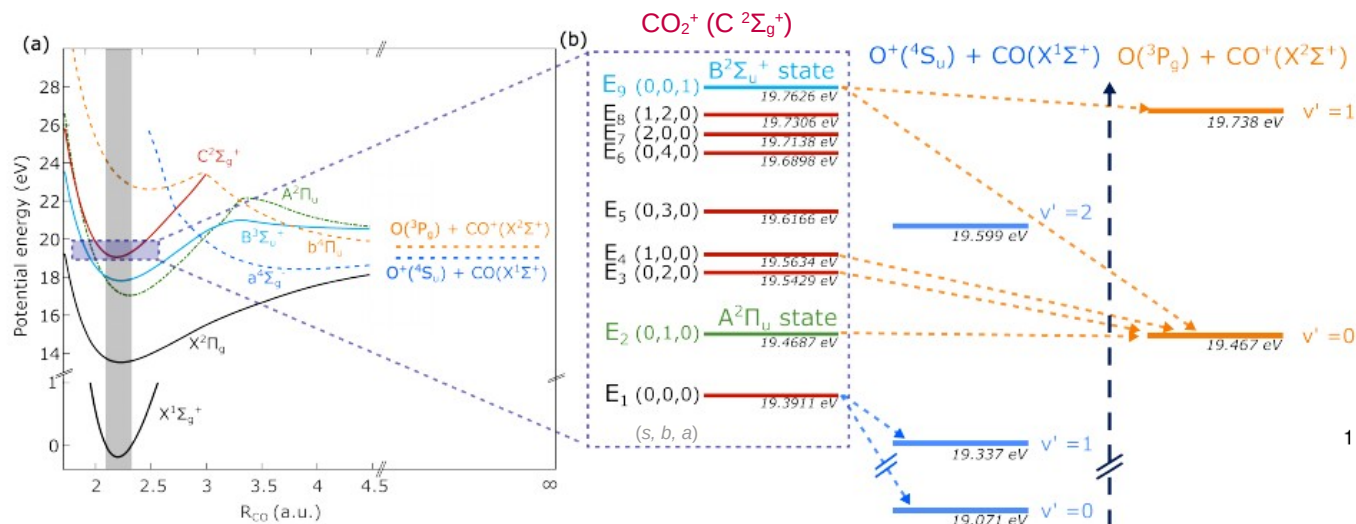
CO<sub>2</sub><sup>+</sup> B<sup>2</sup>Σ<sub>u</sub><sup>+</sup> (v=0) is stable

CO<sub>2</sub><sup>+</sup> C<sup>2</sup>Σ<sub>g</sub><sup>+</sup> dissociates

- (v=0): CO + O<sup>+</sup>
- (v>0): O + CO<sup>+</sup>



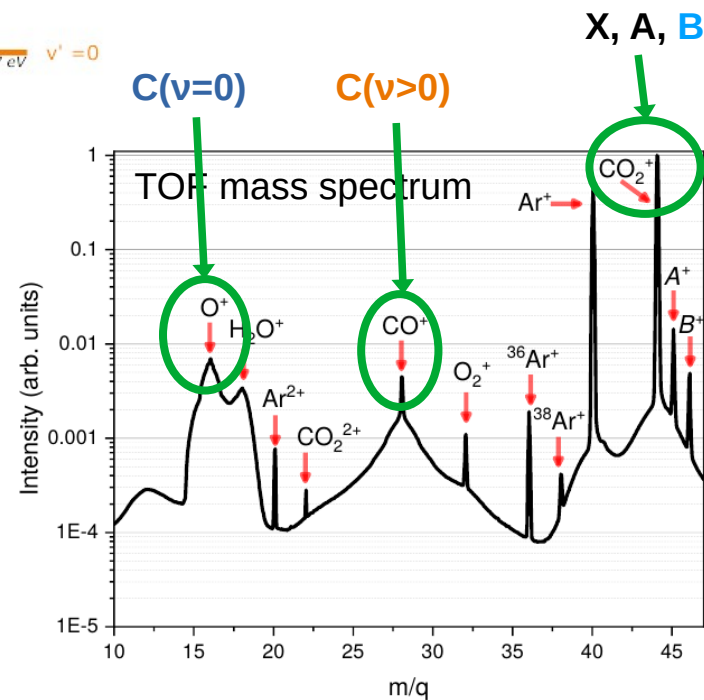
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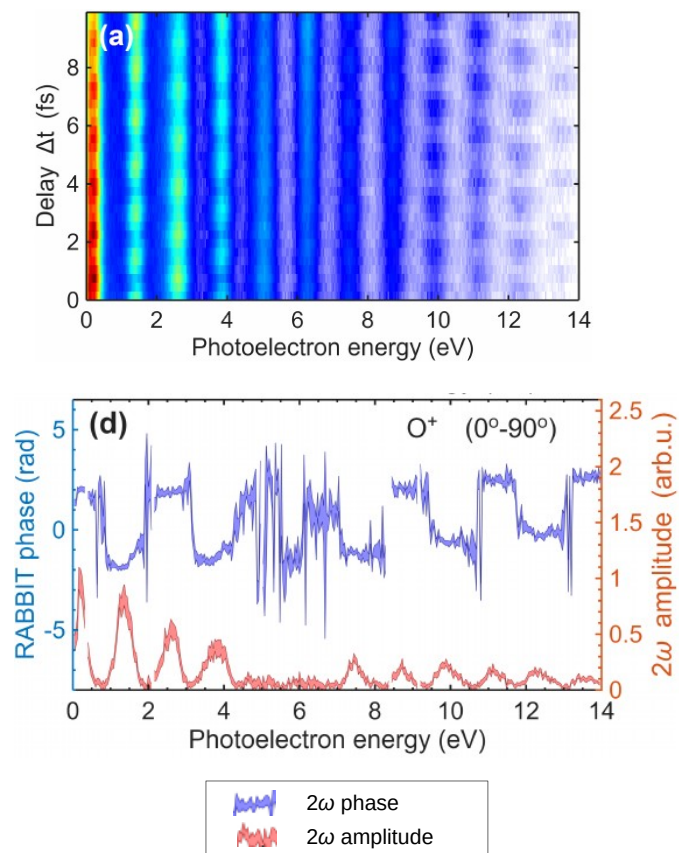
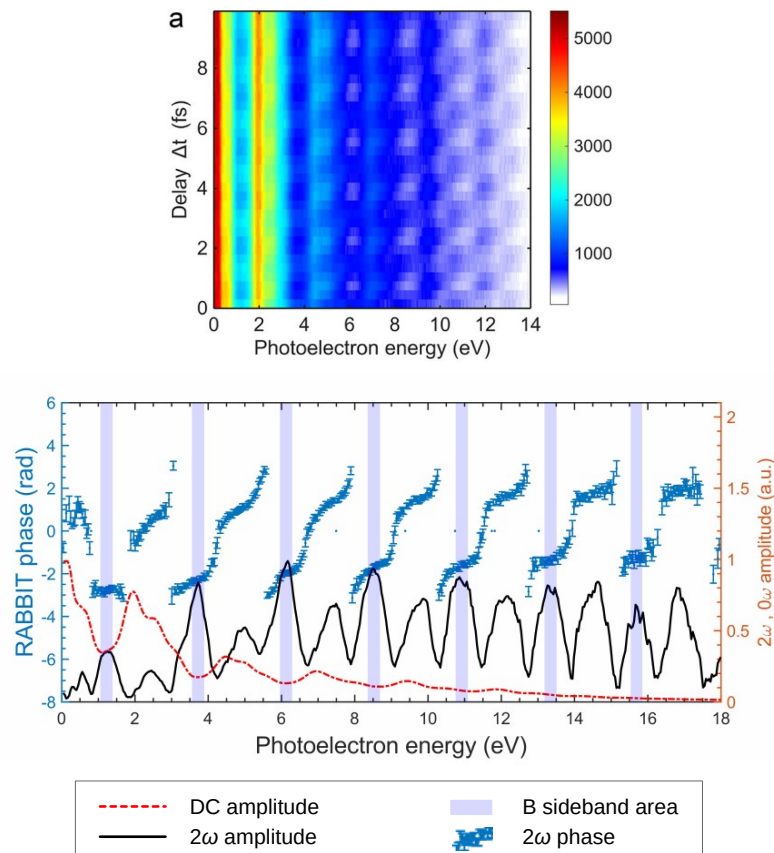
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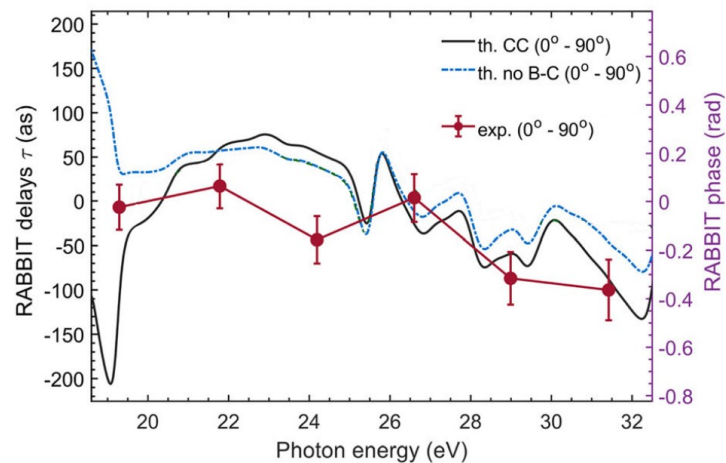
# RABBITT photoelectron spectra

In coincidence with:  $\text{CO}_2^+ \sim \text{X,A,B}$

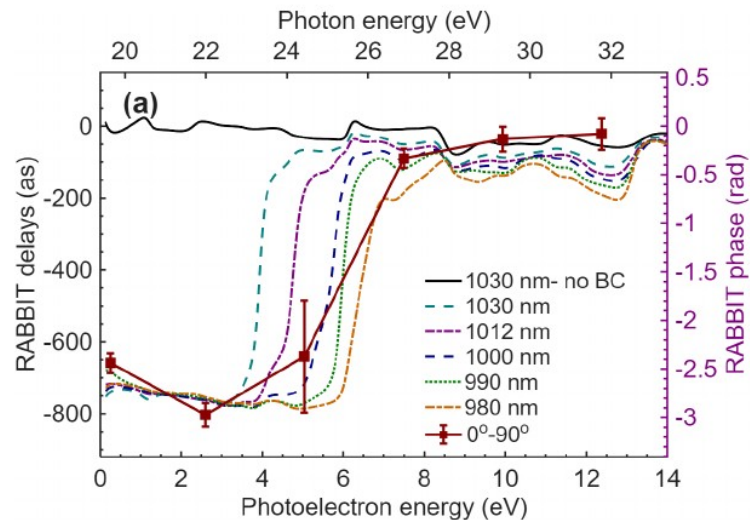
$\text{O}^+ \sim \text{C}(v=0)$



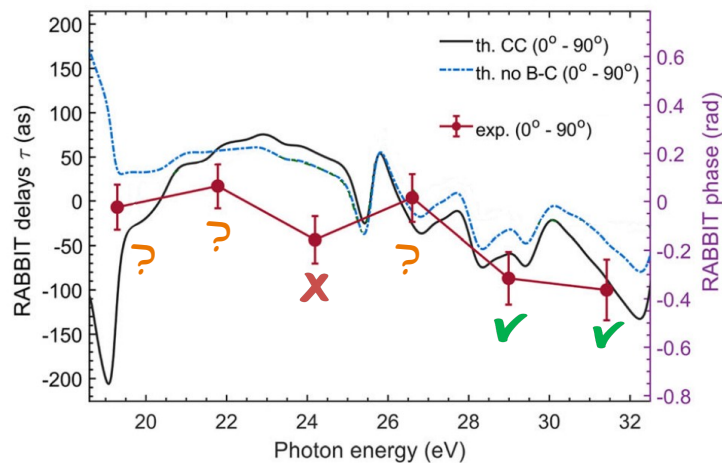
$\text{CO}_2^+ \text{ B } ^2\Sigma_u^+$ , relative to  $\text{Ar}^+(3p)$



$\text{CO}_2^+ \text{ C } ^2\Sigma_g^+$ , relative to  $\text{Ar}^+(3p)$



$\text{CO}_2^+ \mathbf{B} \ ^2\Sigma_u^+$ , relative to  $\text{Ar}^+(3p)$



- ▶ Small effect – and nonuniform at low energies.
- ▶ Theory-experiment agreement improves towards higher energies.
- ▶ Inadequate modelling of autoionizing resonances?
- ▶ Initial-state vibrational motion?

$$M = \langle \Psi_f | (\mathbf{r}_{pe} + \mathbf{D}_{ion}) \cdot \boldsymbol{\epsilon}_{IR} | \Psi_{i+\Omega} \rangle$$

## Pathway interference ( $B$ channel)

$$\tau_R = \frac{1}{2\omega_{\text{IR}}} \arg M_{\text{XUV}+\text{IR}}^* M_{\text{XUV}-\text{IR}}$$

$$\begin{aligned} M_B &= \langle \Psi_B | (\mathbf{r}_{\text{pe}} + \mathbf{D}_{\text{ion}}) \cdot \boldsymbol{\epsilon}_{\text{IR}} | \Psi_{i+\Omega} \rangle \\ &= M_{\text{pe}} + \delta M_{\text{ion}} \end{aligned}$$

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### High-energy limit (mol. frame)

$$M_{B,\text{pe}} \approx \frac{ik}{\omega_{\text{IR}}^2} d_B$$

$$\delta M_{B,\text{ion}} \approx \frac{1}{\Delta_{BC} - \omega_{\text{IR}}} d_C \quad \Delta_{BC} = E_C - E_B$$

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$$\rightarrow \delta \tau_{ii} \approx \frac{1}{2k} \frac{\omega_{\text{IR}}}{\Delta_{BC} - \omega_{\text{IR}}} \frac{\mathbf{D}_{BC} \cdot \boldsymbol{\epsilon}_{\text{IR}}}{\mathbf{k} \cdot \boldsymbol{\epsilon}_{\text{IR}}} \sqrt{\frac{\sigma_C}{\sigma_B}} \cos \arg d_B^* d_C$$

## Pathway interference (B channel)

$$\tau_R = \frac{1}{2\omega_{\text{IR}}} \arg M_{\text{XUV}+\text{IR}}^* M_{\text{XUV}-\text{IR}}$$

⇒ Coupling delay highly sensitive to relation of  $\Delta_{BC}$  and  $\omega_{\text{IR}}$ .

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$$= M_{\text{pe}} + \delta M_{\text{ion}}$$

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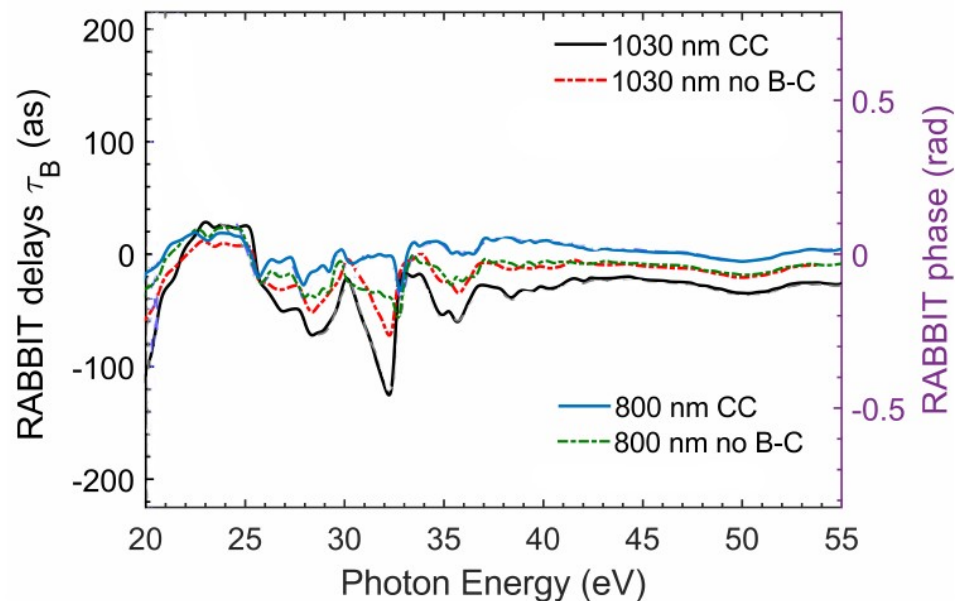
⇒ Sign of  $\tau_{ii}$  depends on sign of  $\Delta_{BC} - \omega_{\text{IR}}$ .

## High-energy limit (mol. frame)

$$M_{B,\text{pe}} \approx \frac{ik}{\omega_{\text{IR}}^2} d_B$$

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## Pathway interference (C channel)

$$\tau_R = \frac{1}{2\omega_{\text{IR}}} \arg M_{\text{XUV}+\text{IR}}^* M_{\text{XUV}-\text{IR}}$$

$$M_C = \langle \Psi_C | (\mathbf{r}_{\text{pe}} + \mathbf{D}_{\text{ion}}) \cdot \boldsymbol{\epsilon}_{\text{IR}} | \Psi_{i+\Omega} \rangle$$

$$= M_{\text{pe}} + \delta M_{\text{ion}}$$

⇒ Coupling delay highly sensitive to relation of  $\Delta_{BC}$  and  $\omega_{\text{IR}}$ .

⇒ Sign of  $\tau_{ii}$  depends on sign of  $\Delta_{BC} - \omega_{\text{IR}}$ .

⇒ Destructive interference for comparable  $M_{\text{pe}}$  and  $\delta M_{\text{ion}}$ .

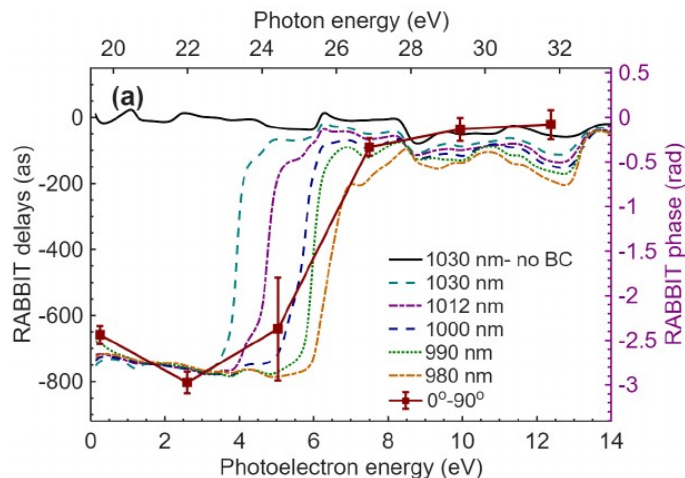
## High-energy limit (mol. frame)

$$M_{C,\text{pe}} \approx \frac{ik}{\omega_{\text{IR}}^2} d_C$$

$$\delta M_{C,\text{ion}} \approx \frac{1}{\Delta_{BC} - \omega_{\text{IR}}} d_B \quad \Delta_{BC} = E_C - E_B$$

$$\rightarrow \delta \tau_{ii} \approx \frac{1}{2k} \frac{\omega_{\text{IR}}}{\Delta_{BC} - \omega_{\text{IR}}} \dots \text{etc ...}$$

## CO<sub>2</sub><sup>+</sup> C 2<sup>2</sup>Σ<sub>g</sub><sup>+</sup>, relative to Ar<sup>+</sup>(3p)



## Pathway interference (C channel)

$$\tau_R = \frac{1}{2\omega_{\text{IR}}} \arg M_{\text{XUV}+\text{IR}}^* M_{\text{XUV}-\text{IR}}$$

$$M_C = \langle \Psi_C | (\mathbf{r}_{\text{pe}} + \mathbf{D}_{\text{ion}}) \cdot \boldsymbol{\epsilon}_{\text{IR}} | \Psi_{i+\Omega} \rangle$$

$$= M_{\text{pe}} + \delta M_{\text{ion}}$$

⇒ Coupling delay highly sensitive to relation of  $\Delta_{BC}$  and  $\omega_{\text{IR}}$ .

⇒ Sign of  $\tau_{ii}$  depends on sign of  $\Delta_{BC} - \omega_{\text{IR}}$ .

⇒ Destructive interference for comparable  $M_{\text{pe}}$  and  $\delta M_{\text{ion}}$ .

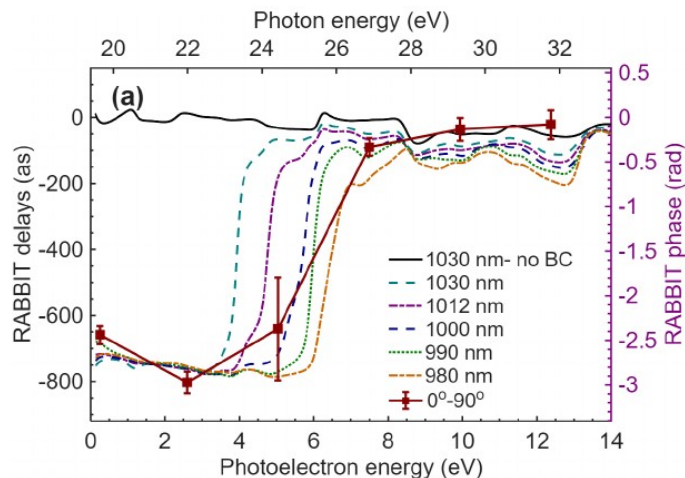
## High-energy limit (mol. frame)

$$M_{C,\text{pe}} \approx \frac{ik}{\omega_{\text{IR}}^2} d_C$$

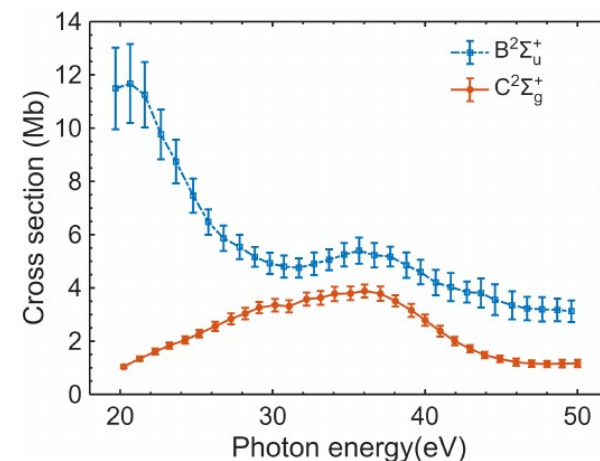
$$\delta M_{C,\text{ion}} \approx \frac{1}{\Delta_{BC} - \omega_{\text{IR}}} d_B \quad \Delta_{BC} = E_C - E_B$$

$$\rightarrow \delta \tau_{ii} \approx \frac{1}{2k} \frac{\omega_{\text{IR}}}{\Delta_{BC} - \omega_{\text{IR}}} \dots \text{etc} \dots$$

## CO<sub>2</sub><sup>+</sup> C<sup>2</sup>Σ<sub>g</sub><sup>+</sup>, relative to Ar<sup>+</sup>(3p)



## Cross sections B vs C

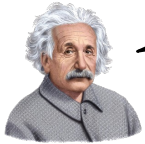


## Three-path RABBITT

Transition in residual ion affects delay of photoelectrons...?

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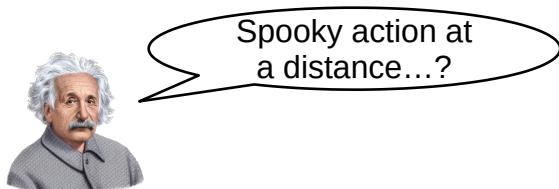
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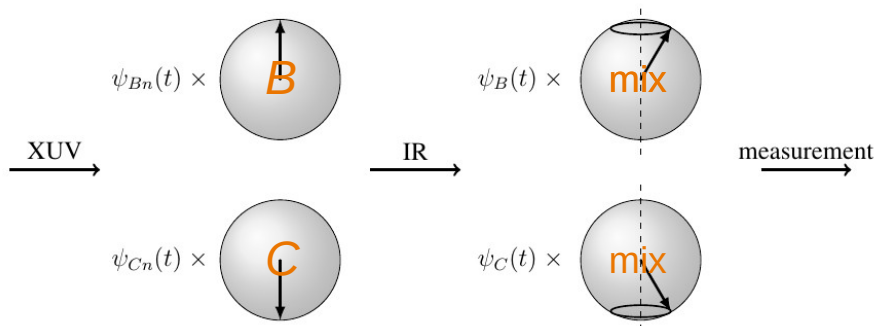
Spooky action at  
a distance...?

# Three-path RABBITT

Transition in residual ion affects delay of photoelectrons...?



Actually: *entanglement*



Projection on *B* or *C* (after IR) **mixes** photoelectrons associated with the original *B* and *C* channels.

# Entanglement

Degree of entanglement affects delay of photoelectrons

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State after XUV ionization:

$$\Psi_n \sim \psi_B^{\text{pe}} \Phi_B^{\text{ion}} + \psi_C^{\text{pe}} \Phi_C^{\text{ion}}$$

$$\rho = |\Psi_n\rangle\langle\Psi_n|$$



Reduced DM of the ion subsystem:

$$\rho_{\text{ion}} = \text{Tr}_{\text{pe}} \rho \sim \frac{1}{\sigma_B + \sigma_C} \begin{pmatrix} \sigma_B & 0 \\ 0 & \sigma_C \end{pmatrix}$$



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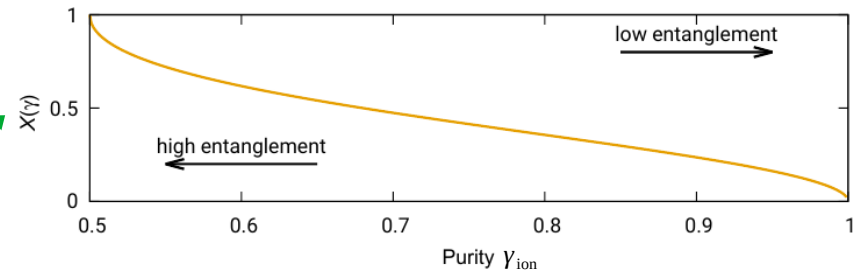
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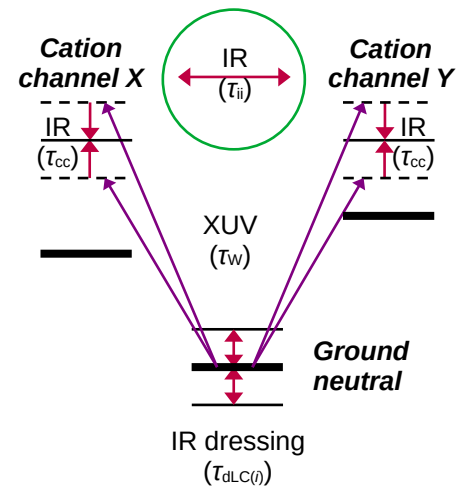
$\chi(\gamma_{\text{ion}})$



⇒ Coupling delay high for *highly entangled* intermediate states.

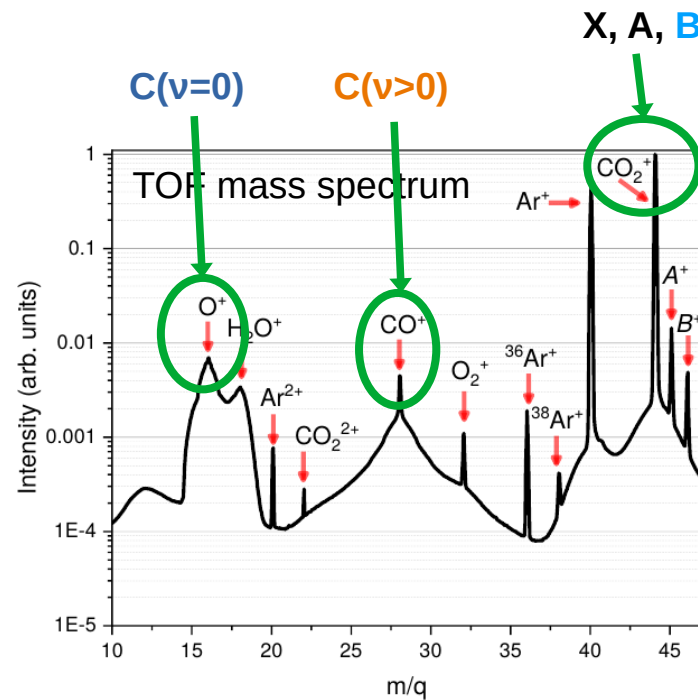
## Conclusion

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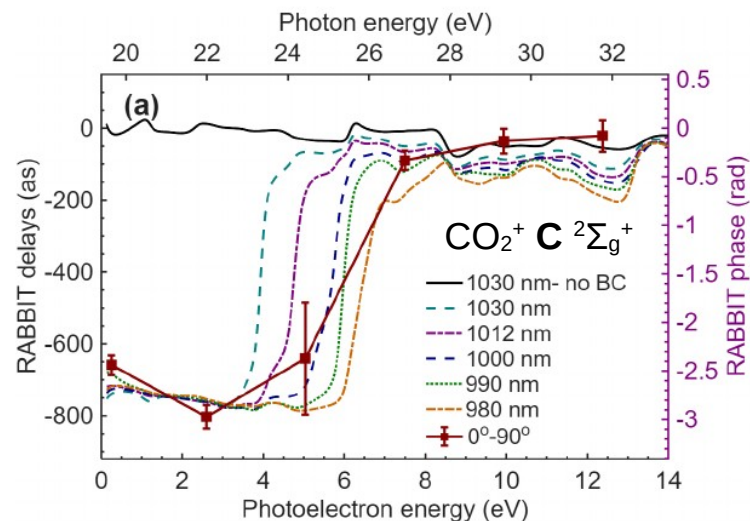
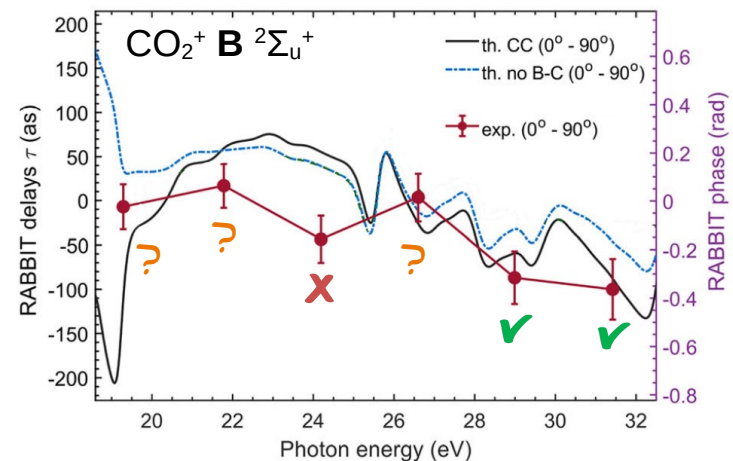
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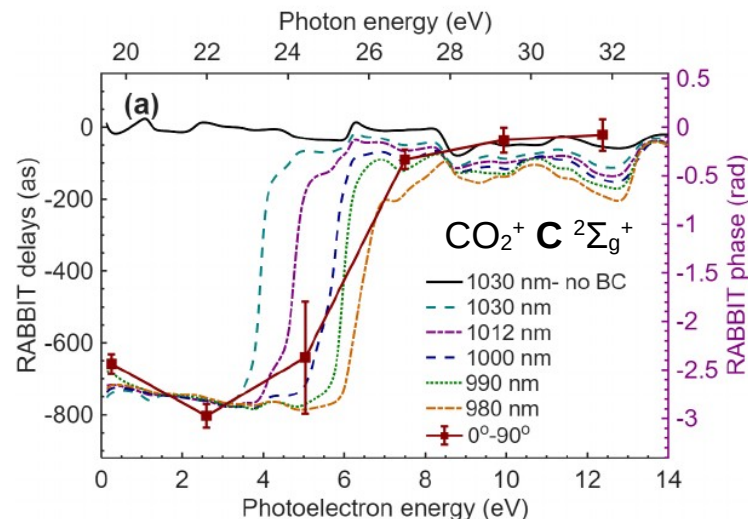
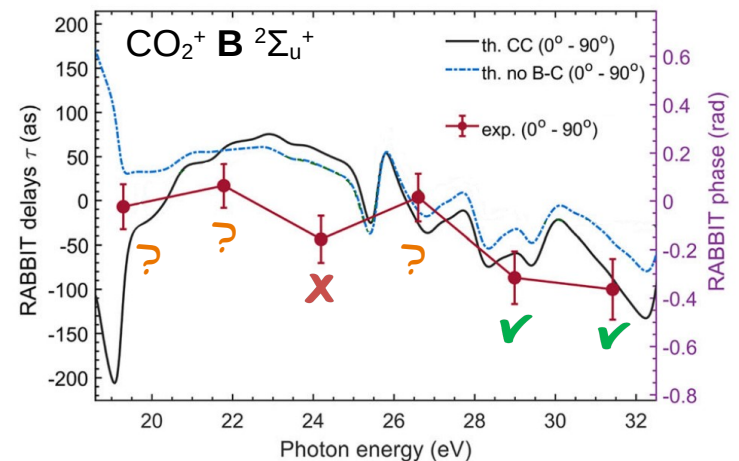
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**Thank you for attention!**

