ELISS2023

ELI Summer School | 29 Aug – 1 Sep 2023

Dolní Břežany, Czech Republic

Strong field processes in ultra-intense laser fields

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ELI Beamlines

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Dolní Břežany, Czech Republic

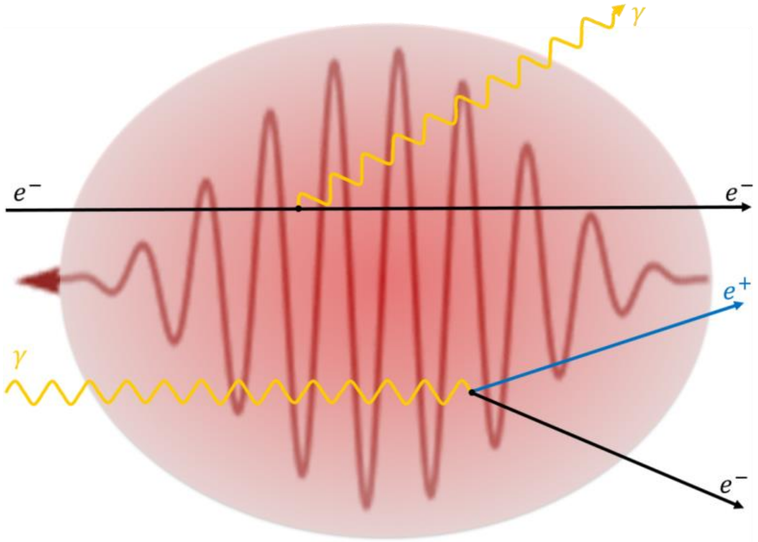
eli

IMPULSE

IMPULSE is funded by the European Union's Horizon 2020 programme under grant agreement No. 871161

Why study strong-field QED?

- Processes in plasma around pulsars and magnetars
- Other extreme astronomical environments – accretion disks of black holes, quasars, cosmic radiation
- QED plasma present in early universe during BBN
- Heavy ion collisons
- Electron beam interaction with aligned crystals
- Fundamental limits of quantum theory
- Nature of vacuum itself and quantum fluctuations



From www.mpi-hd.mpg.de

Lasers are becoming a great tool to probe such environments!

Review articles:

- Extremely high-intensity laser interactions with fundamental quantum systems, Di Piazza et al., Rev. Mod. Phys. **84**, (2012) 1177
- Charge particle motion and radiation in strong electromagnetic fields, Gonoskov et al., Rev. Mod. Phys. **94**, (2022) 045001
- Advances in QED with intense background fields, Fedotov et al., Phys. Rep. **1010**, (2023) 1-138

Electromagnetic field scale

Schwinger critical field: does work equal to the electron rest energy mc^2 over distance of reduced Compton length $\hbar/mc$

$$E_{cr} = \frac{m^2c^3}{e\hbar} = 1.323 \times 10^{18}\ \mathrm{V/m}$$

Pb-Pb 3TeV collisions $E \sim 10^{24}$ V/m

$$I_{cr} = 4.6 \times 10^{29}\ \mathrm{W/cm^2}$$

Laser world record @ KoRELs (2021) $\sim 10^{23}\ \mathrm{W/cm^2}$

Equivalently: $$B_{cr} = \frac{E_{cr}}{c} = 4.41 \times 10^{9}\ \mathrm{T}$$

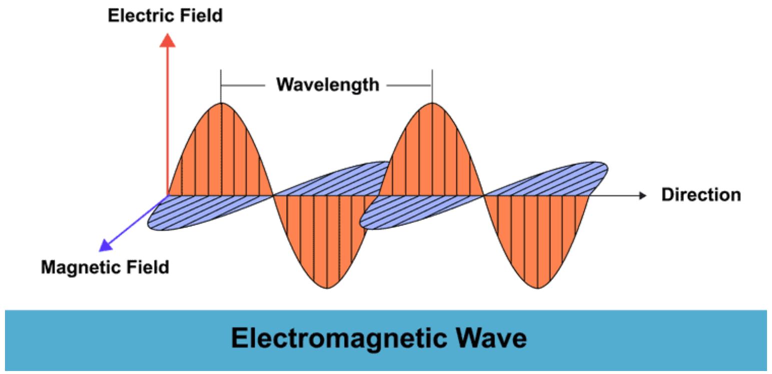
Magnetar field $\sim 10^{11}$ T

Field invariants (observer independent):

$$\mathcal{F} = \frac{1}{4E_{cr}^2} \mathrm{F}_{\mu\nu}\mathrm{F}^{\nu\mu} = \frac{1}{2E_{cr}^2}(\boldsymbol{E}^2 - c^2\boldsymbol{B}^2)$$

$$\mathcal{G} = \frac{1}{4E_{cr}^2} \mathrm{F}_{\mu\nu}\tilde{F}^{\nu\mu} = \frac{c\boldsymbol{B} \cdot \boldsymbol{E}}{E_{cr}^2}$$

Both zero in a plane wave field!



Collision invariants

$$\chi_e = \frac{\sqrt{-\left(F_{\mu\nu}p^\nu\right)^2}}{mcE_{cr}} = \left.\frac{E_L}{E_{cr}}\right|_{R.F.} = 5.9\times10^{-2}\mathcal{E}[\mathrm{GeV}]\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}$$

Field strength in particle’s rest frame

$$\chi_\gamma = \frac{\sqrt{-\left(F_{\mu\nu}k^\nu\right)^2}}{mcE_{cr}}$$

Equivalent for massless photons

Normalized EM wave amplitude / unitless vector potential:

For plane wave field

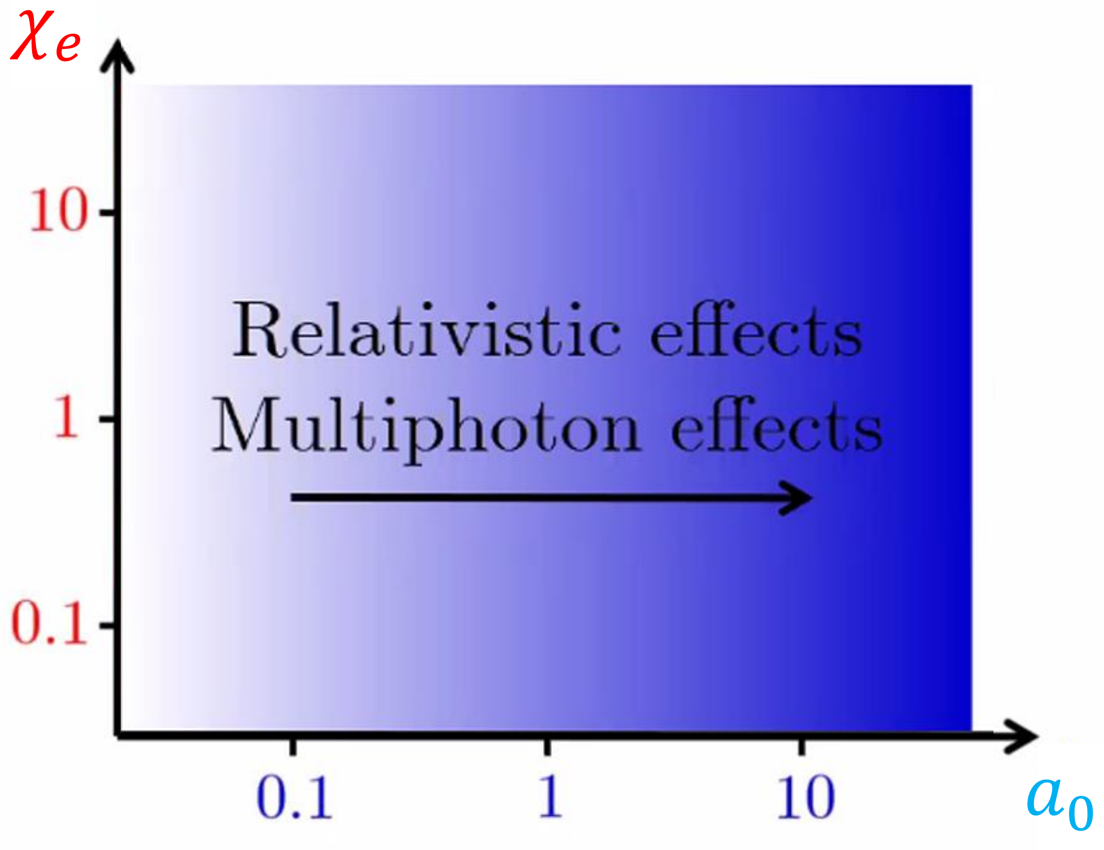
$$a_0 = \frac{1}{mc^2}\frac{e\sqrt{-\left(F_{\mu\nu}p^\nu\right)^2}}{\kappa\cdot p} = \frac{e\sqrt{-A^2}}{mc} = \frac{eE_L}{mc\omega_L} = 7.5\frac{\sqrt{I_L[10^{20}\ \mathrm{W/cm^2}]}}{\hbar\omega_L[\mathrm{eV}]}$$

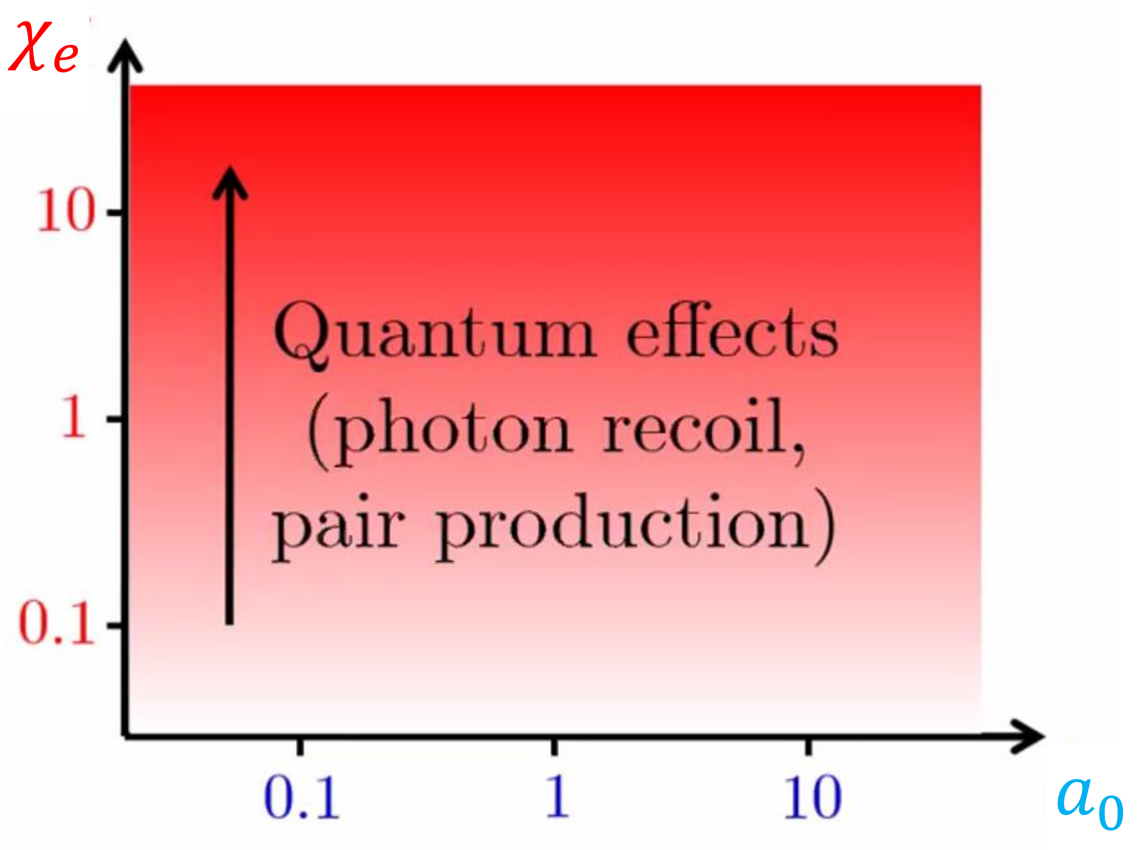
$$u_\perp = \gamma\beta_\perp \approx a_0$$

$a_0 \approx 1$ threshold for relativistic and nonlinear effects. Number of absorbed photons $n \approx a_0^3$

Schwinger field: $$a_{cr} = \frac{eE_{cr}}{mc\omega} = \frac{mc^2}{\hbar\omega_L} = 4.1\times10^5\lambda^{-1}[\mu\mathrm{m}]$$

Reminder: a_0 – field strength, χ_e – what field the particle sees in its rest frame





Courtesy A. Di Piazza

$a_0 \approx 1$:
$$I_{rel} = 1.37 \times 10^{18} \lambda^{-2}[\mu\mathrm{m}]\ \mathrm{W/cm^2}$$

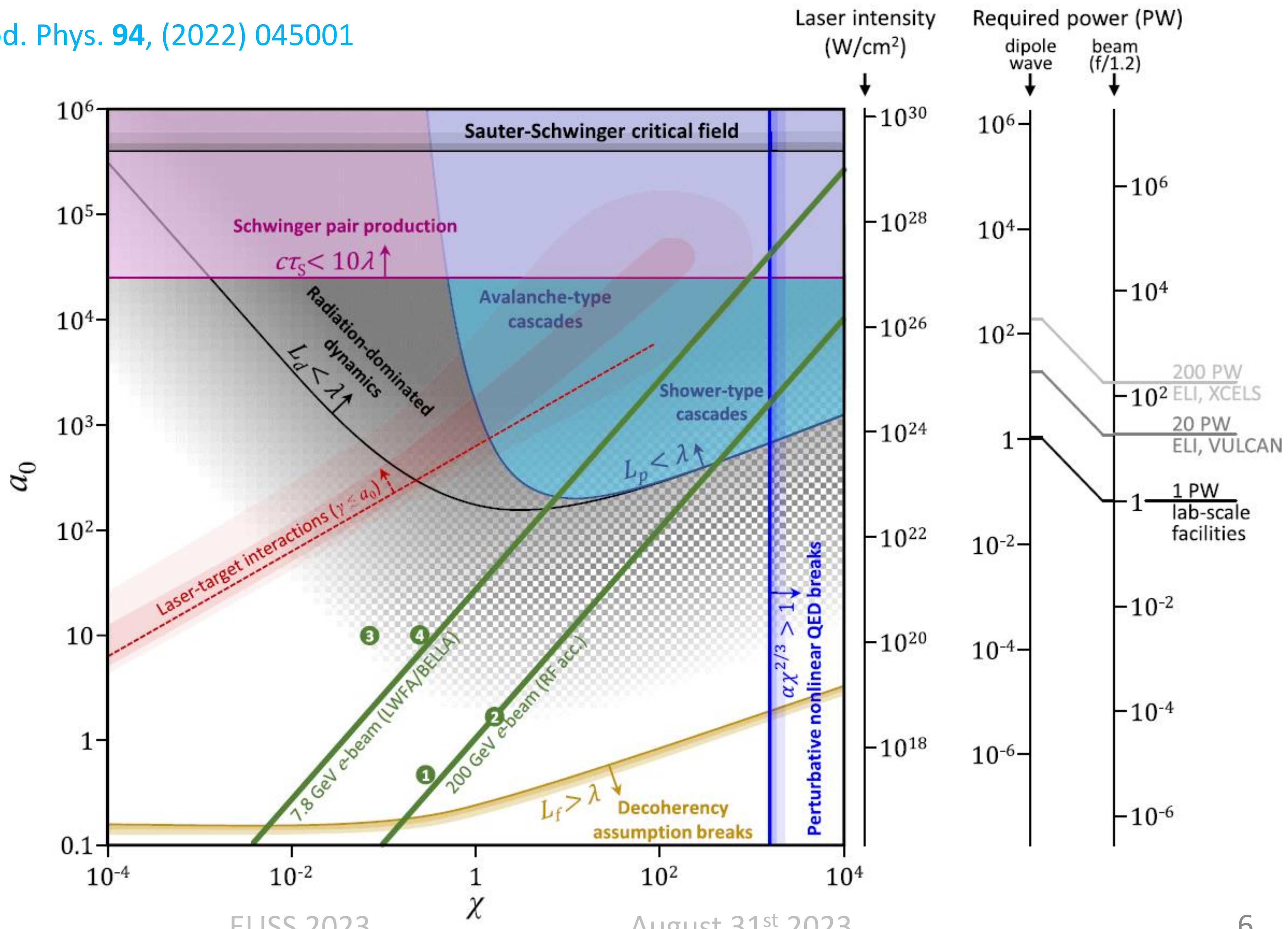
$\chi_e \approx 1$:
$$I_{QED} = 5.75 \times 10^{23} \lambda^{-1}[\mu\mathrm{m}]\mathrm{W/cm^2}$$

$$I_{\mathrm{record}} = 1.1 \times 10^{23}\ \mathrm{W/cm^2}$$

Schwinger field: $I_{cr} = 4.6 \times 10^{29}\ \mathrm{W/cm^2}$

Gonoskov et al., Rev. Mod. Phys. **94**, (2022) 045001

Reminder:
a_0 – field strength,
χ – what field the particle sees in its rest frame



Classical radiation

Two separate theories of electromagnetism:

- One which from prescribed fields calculates the motion of charged particles.
- The other which from given sources of charges and currents calculates the fields

Jackson, *Classical Electrodynamics 3rd edition*, John Wiley & Sons, Hoboken, NJ, (1999).

Classical motion in external fields – Lorentz force:

External field $F^{\mu\nu}$

$$\dot{u}^\mu = \frac{e}{m} F^{\mu\nu} u_\nu$$

Radiation from moving charges – Maxwell equations – solution: Lienard-Wiechert potentials

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \boldsymbol{n} \times (\boldsymbol{n} \times \dot{\boldsymbol{r}}(t)/c) e^{i\omega(t - \boldsymbol{n}\cdot\boldsymbol{r}(t)/c)} dt\right|^2$$

But particle emitting radiation must change its trajectory! Trajectory $\boldsymbol{r}(t)$

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi\varepsilon_0 mc^3} = 6.3 \times 10^{-24}\text{s}$$

Radiation reaction!!

Lorentz-Abraham-Dirac (1937)

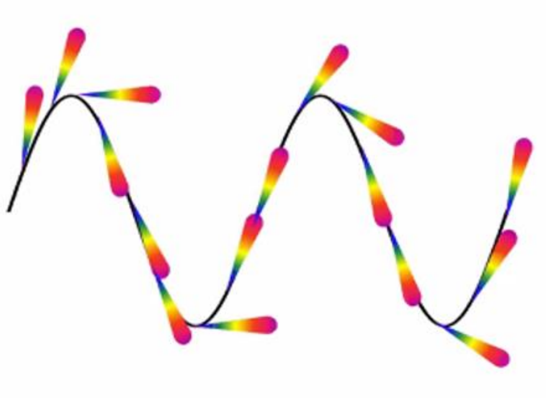
$$\dot{u}^\mu = \frac{e}{m} F^{\mu\nu} u_\nu + \tau_0 \left(\ddot{u}^\mu + \frac{\dot{u}^2}{c^2} u^\mu \right)$$

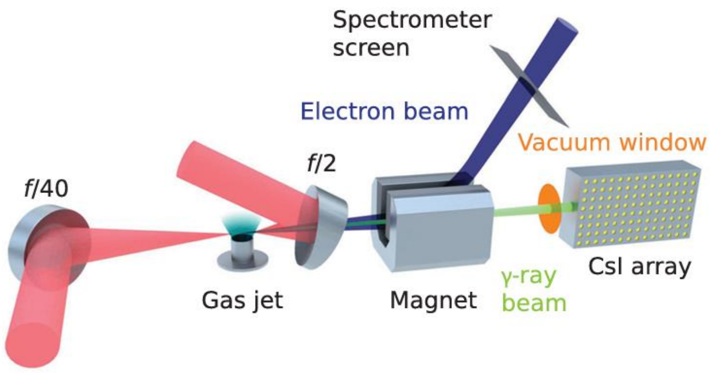
Quantum radiation

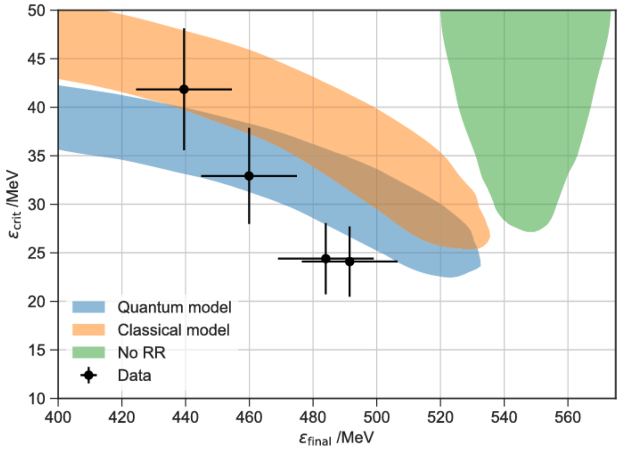
- Described by quantum field theory – specifically by quantum electrodynamics (QED)
- Typical energy of emitted photons comparable to the energy of the particles. In classical case nothing prevents photons with energy higher than the energy of the particle.
- Discrete instantaneous recoils that occur along the particle trajectory at probabilistically determined intervals

  Opposite to continuous force in classical case!!
- Stochasticity
- Straggling (electrons have a chance of not-emitting resulting into higher energy electrons than expected)
- Quenching – extreme case – no emission

Classical limit:







Cole et al., PRX 8 (2018) 011020;
Poder et al., PRX 8 (2018) 031004
Astra-Gemini

Furry picture

Coupling to laser field treated exactly to all orders
Coupling to radiation field perturbative!

External classical field A^{μ}

Quantum relativistic equation for fermions

Dirac equation: $[\gamma\cdot(p-eA)-mc]\psi=0$

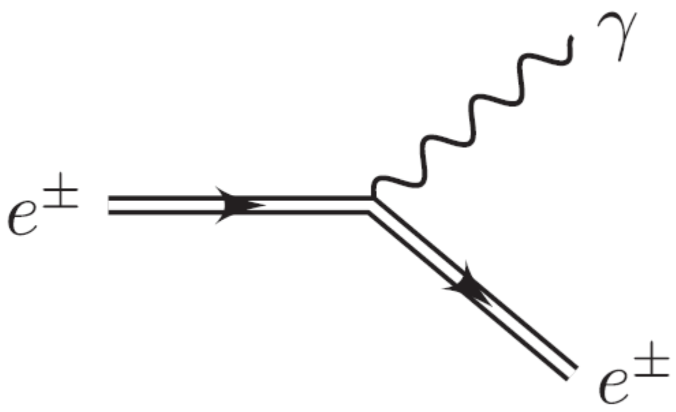
Volkov solution for plane wave external field:

$$\psi_{pr}(x)=\left[1+\frac{ek\cdot\gamma A(\varphi)\cdot\gamma}{2k\cdot p}\right]\mathrm{e}^{-\mathrm{i}S(\varphi)/\hbar}u_{pr}$$

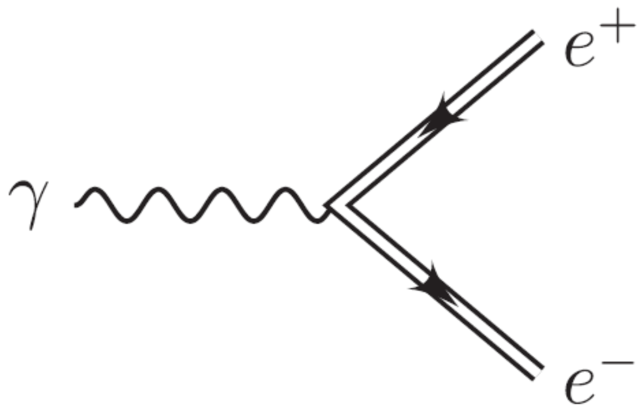
$$S(\varphi)=p\cdot x+\int_{-\infty}^{\varphi}d\varphi'\,\frac{2ep\cdot A(\varphi')-e^2A^2(\varphi')}{2k\cdot p}$$

Quasiclassical form –
Connection to WKB approximation

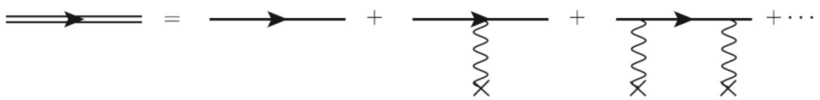
$$\psi(x)=\sqrt{\rho(x)}e^{-iS(x)/\hbar}$$



Compton scattering

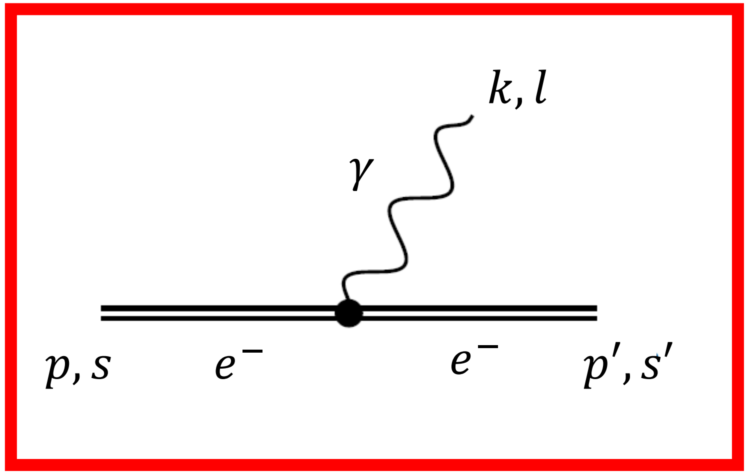


Breit-Wheeler process



“dressed” particle state

QED processes in the Furry picture



Compton scattering

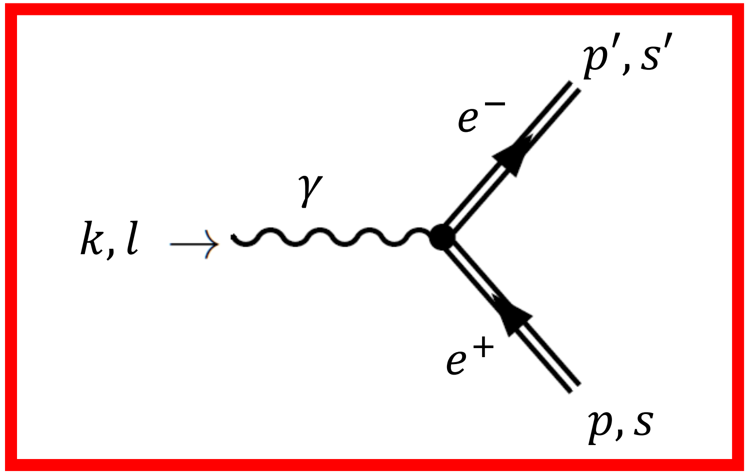
$$S_{fi}^{(e^-\to e^-\gamma)} = -ie\int d^4x\,\overline{U}_{p',s'}^{(out)}(x)\,\frac{\gamma\cdot e_{k,l}e^{ik\cdot x}}{\sqrt{2k_+V_0}}\,U_{p,s}^{(in)}(x)$$

$$dP^{(e^-\to e^-\gamma)} = \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{2}\sum_{l,s,s'}|S_{fi}|^2$$

Breit-Wheeler pair production

$$S_{fi}^{(\gamma\to e^-e^+)} = -ie\int d^4x\,\overline{U}_{p',s'}^{(out)}(x)\,\frac{\gamma\cdot e_{k,l}e^{-ik\cdot x}}{\sqrt{2k_+V_0}}\,V_{p,s}^{(out)}(x)$$

$$dP^{(\gamma\to e^-e^+)} = \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{d^3\boldsymbol{p}'}{(2\pi)^3}\frac{1}{2}\sum_{l,s,s'}|S_{fi}|^2$$

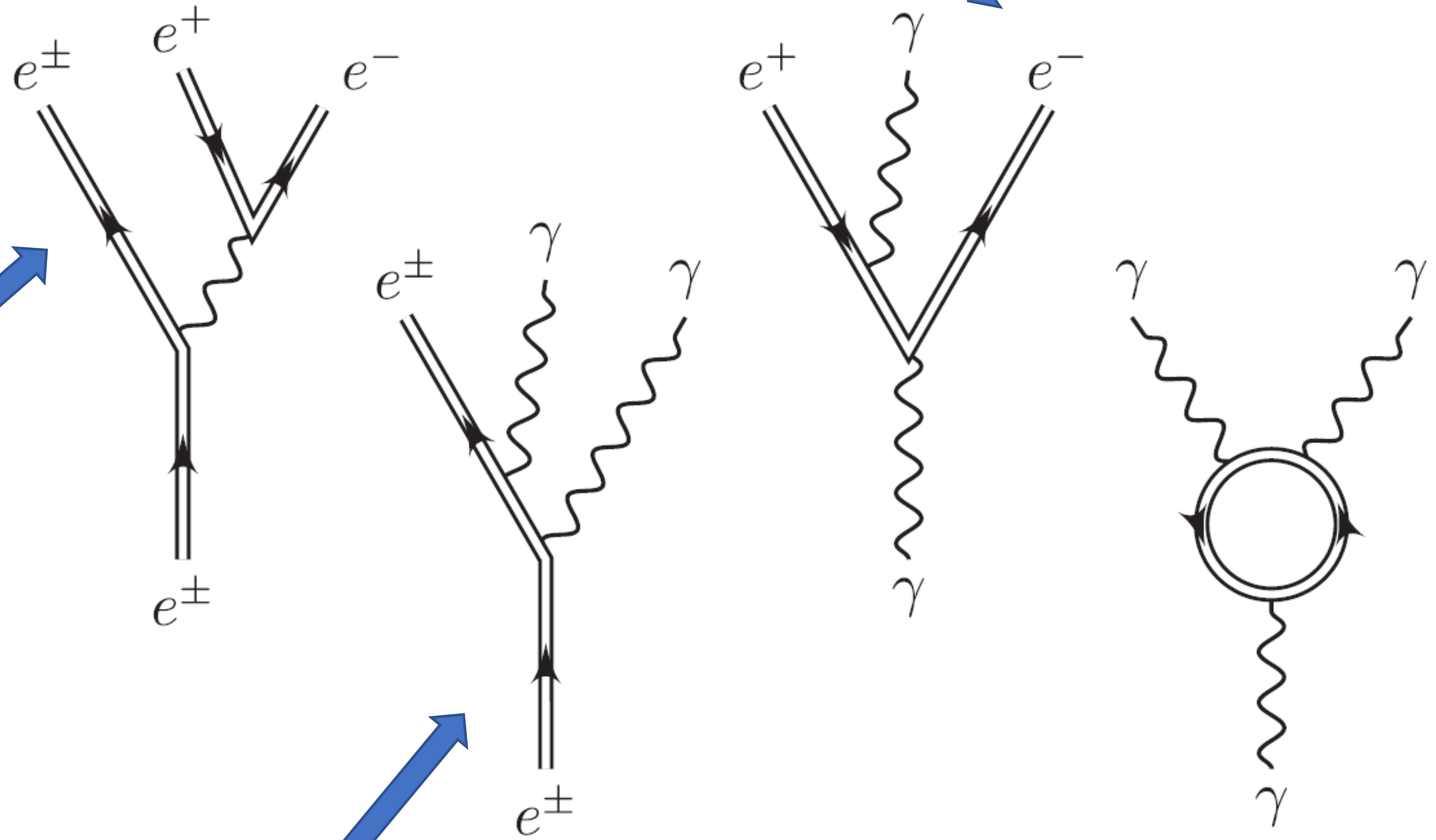


Higher order processes

Phototrident pair creation

Photon splitting

Trident process



Double Compton scattering

Gonoskov et al., Rev. Mod. Phys. **94**, (2022) 045001

Locally-constant-crossed field approximation

Allows calculating strong-field probabilities for arbitrary fields using known results for constant crossed fields

Crossed:

Ultrarelativistic particles see in their rest frame a field (Lorentz transformation):

$$\boldsymbol{E}'_\perp = \gamma(\boldsymbol{E}_\perp + \boldsymbol{\beta} \times \boldsymbol{B}_\perp) \qquad E'_\parallel = E_\parallel$$

$$\boldsymbol{B}'_\perp = \gamma(\boldsymbol{B}_\perp - \boldsymbol{\beta} \times \boldsymbol{E}_\perp) \qquad B'_\parallel = B_\parallel$$

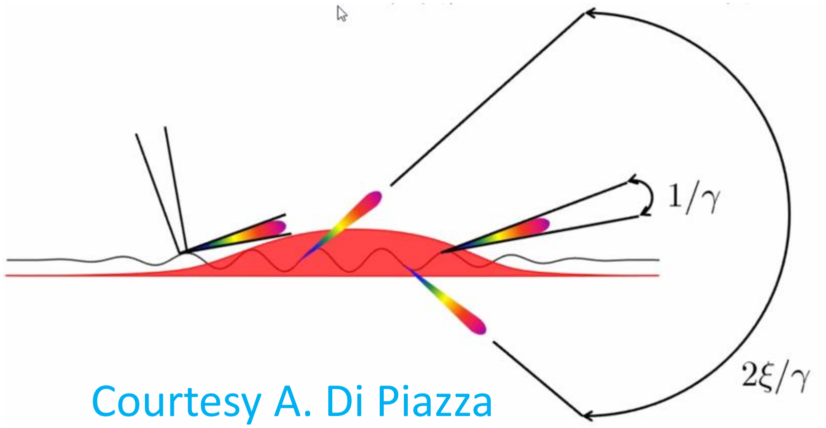
This means that the transverse components are amplified and for high γ become dominant. Moreover

$$\boldsymbol{E}' \cdot \boldsymbol{B}' = \boldsymbol{E} \cdot \boldsymbol{B} \qquad \boldsymbol{E}'^2 - \boldsymbol{B}'^2 = \boldsymbol{E}^2 - \boldsymbol{B}^2$$

are not amplified so for sufficiently fast particles the field is pretty much orthogonal $\frac{|\boldsymbol{E}' \cdot \boldsymbol{B}'|}{\boldsymbol{E}'^2 + \boldsymbol{B}'^2} \sim \frac{1}{\gamma^2}$

Same applies to $\frac{|\boldsymbol{E}'^2 - \boldsymbol{B}'^2|}{\boldsymbol{E}'^2 + \boldsymbol{B}'^2} \sim \frac{1}{\gamma^2}$

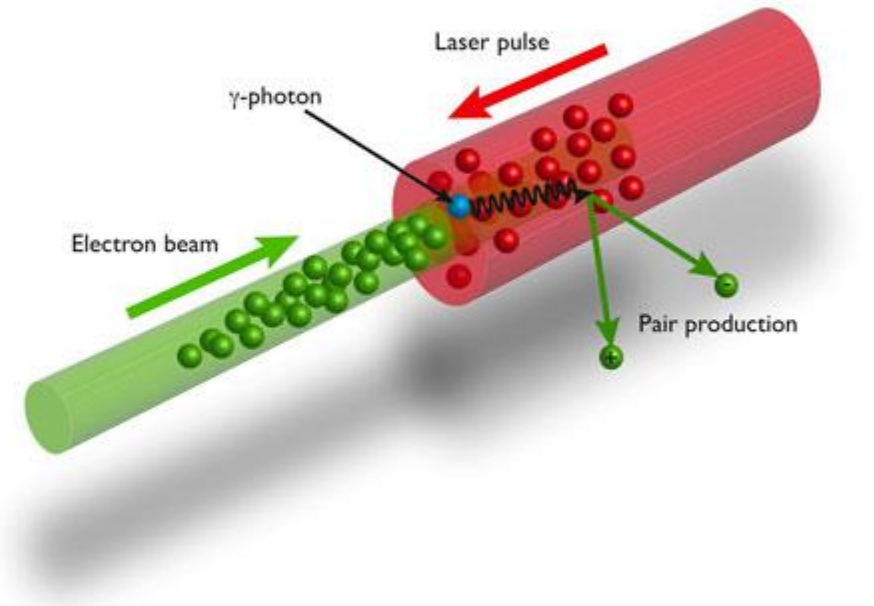
Locally constant:



Courtesy A. Di Piazza

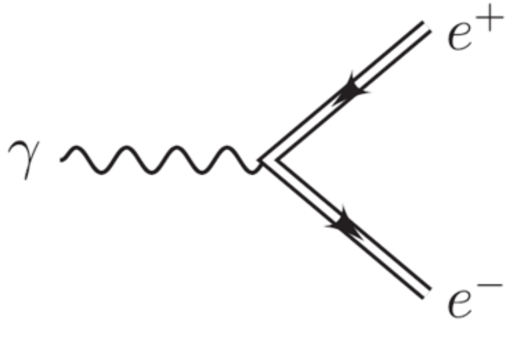
Emission in a thin $1/\gamma$ cone originates from a very small part of the trajectory where the fields can be considered constant

QED processes



https://eli-laser.eu/science-applications/high-fields-physics/

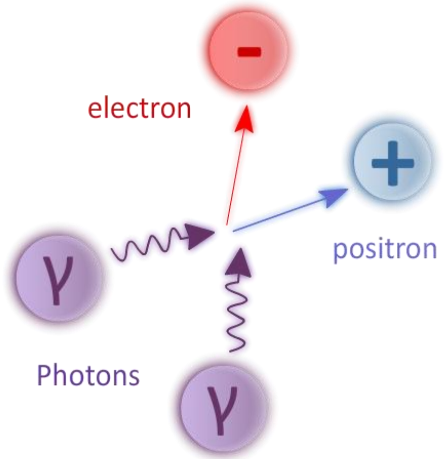
Breit-Wheeler pair production



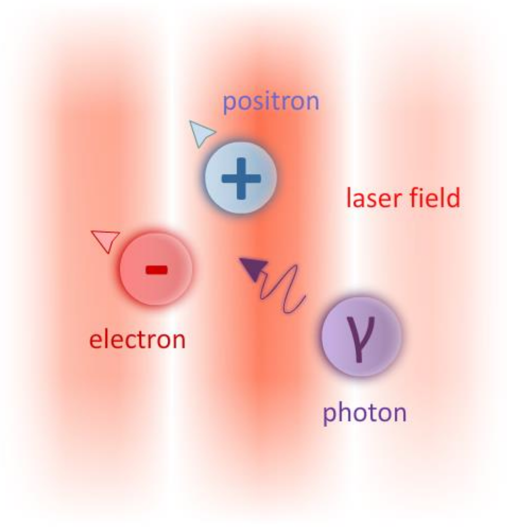
Linear: $\gamma + \gamma' \rightarrow e^+ + e^-$ Yet to be observed

Observed in 1997 at SLAC
50 GeV electrons colliding with 10^{18} W/cm^2 lasers

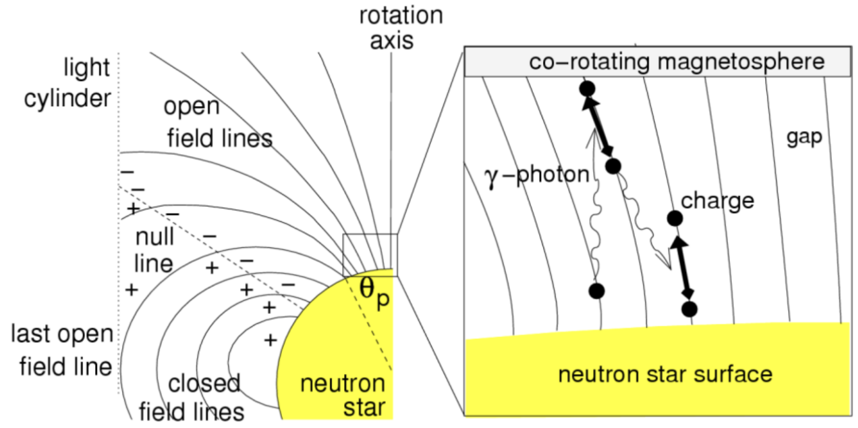
Non-Linear (multiphoton): $\gamma + n\gamma' \rightarrow e^+ + e^-$



Linear BW

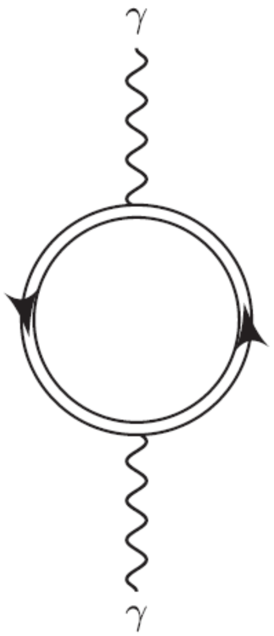


Non-linear BW



Vacuum polarization

Euler-Heisenberg effective Lagrangian (in the lowest order in fields) – interaction of light with itself



$$\mathcal{L} = \frac{1}{2}(E^2 - B^2) + \frac{2\alpha^2}{45m^4}[(E^2 - B^2)^2 + 7(\boldsymbol{E}\cdot\boldsymbol{B})^2] + \cdots$$

Classical part – vacuum Maxwell equations

Interaction part - Maxwell equations in medium

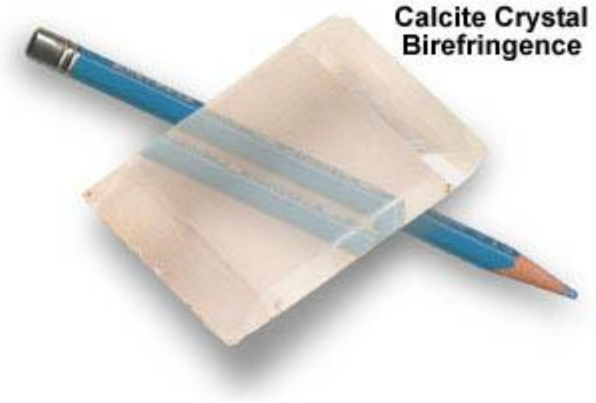
Vacuum becomes polarized and magnetized:

$$\boldsymbol{P} = \frac{\alpha}{180\pi^2 E_{cr}^2}[2(E^2 - B^2)\boldsymbol{E} + 7(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{B}]$$

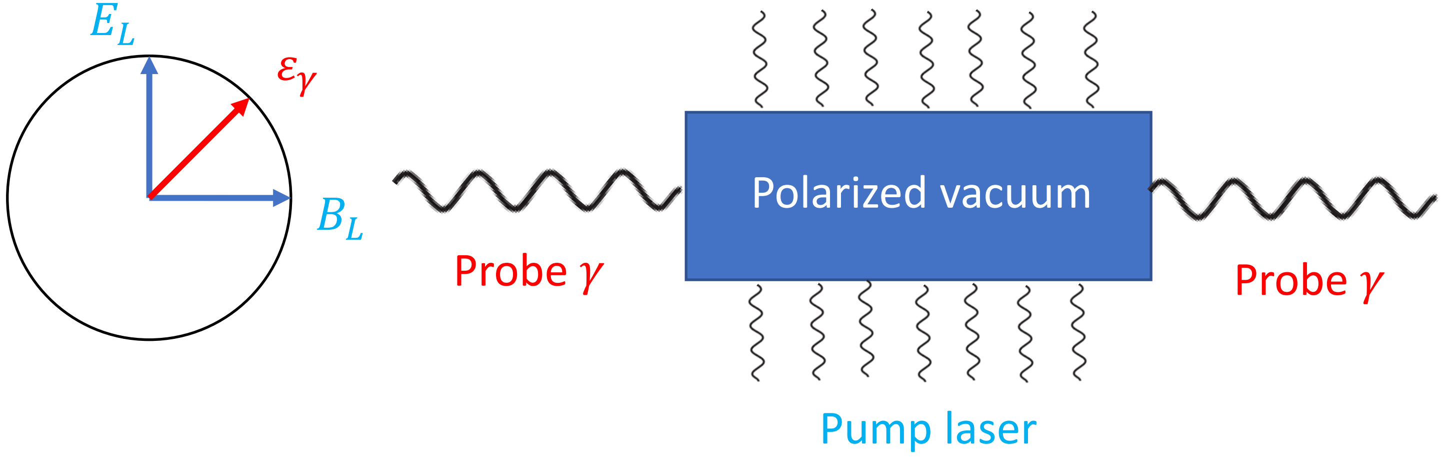
$$\boldsymbol{M} = \frac{\alpha}{180\pi^2 E_{cr}^2}[2(B^2 - E^2)\boldsymbol{B} + 7(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{E}]$$

Different properties in $\boldsymbol{E}$ and $\boldsymbol{B}$ directions

Birefringent medium!



Birefringence measurement

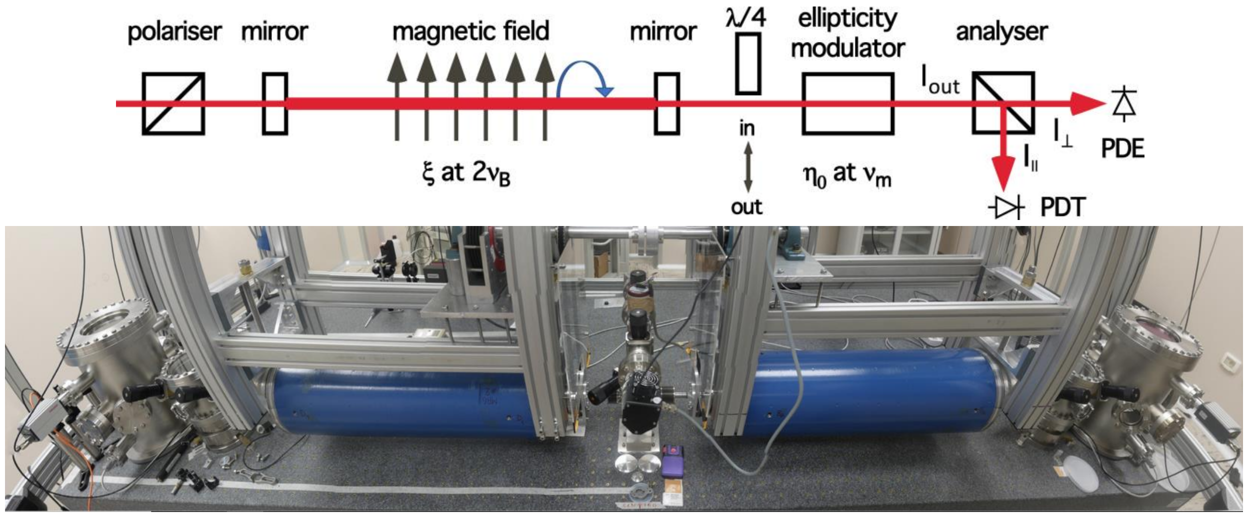


Phase shift between components

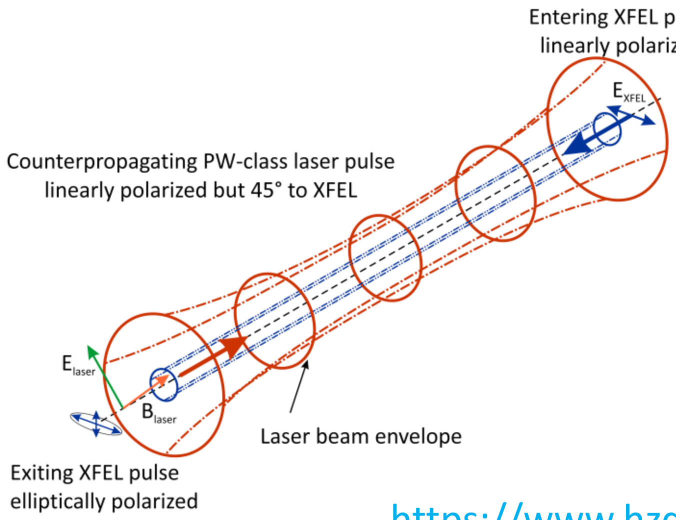
$\Delta\theta$

Elliptical polarization

PVLAS experiment 2001 - ? (constant magnetic field, standing wave)

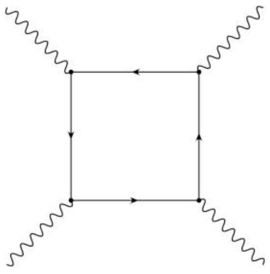


Now with lasers!



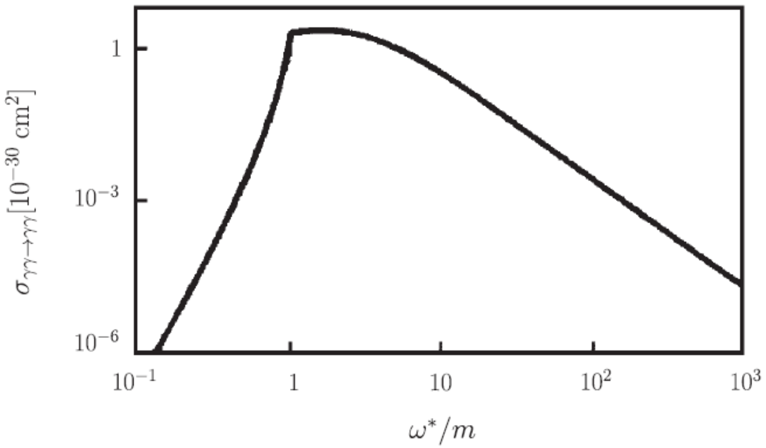
https://www.hzdr.de/

Light-by-light scattering



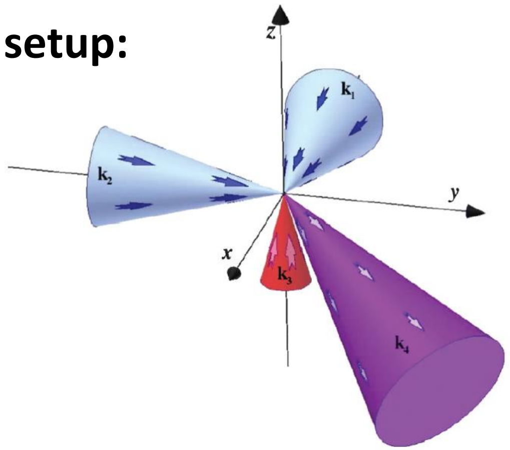
“Box diagram”

cross section



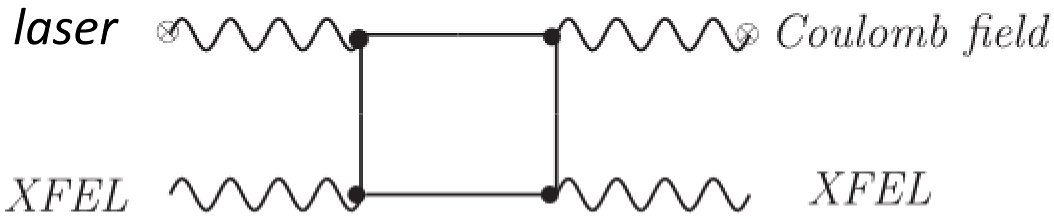
ω^* Center of momentum energy of colliding photons

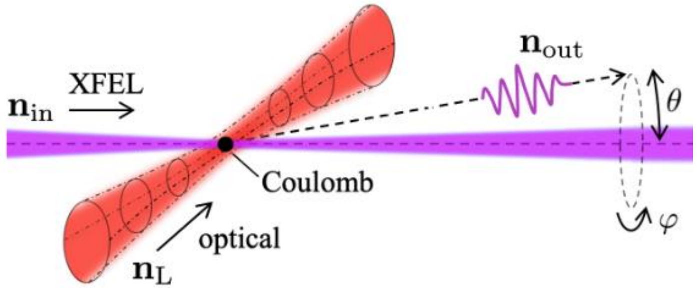
Three laser setup:



Di Piazza et al., Rev. Mod. Phys. **84**, (2012) 1177

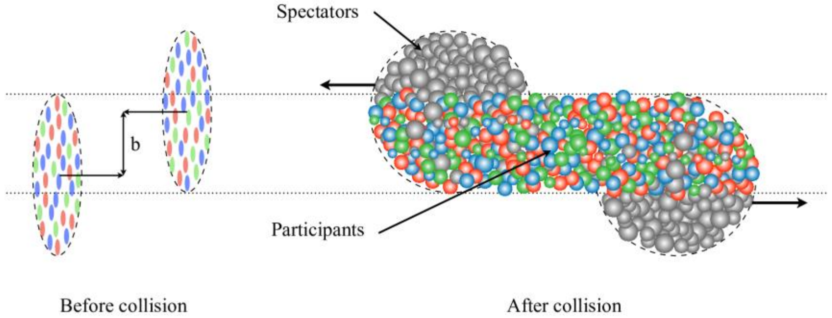
Coulomb assisted





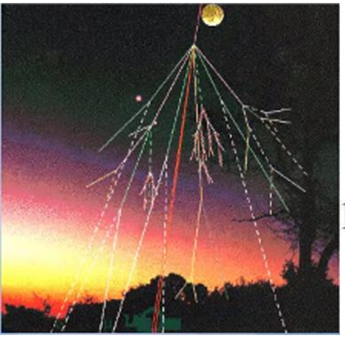
S. Evans, R. Schutzhold, arxiv 2307.08345

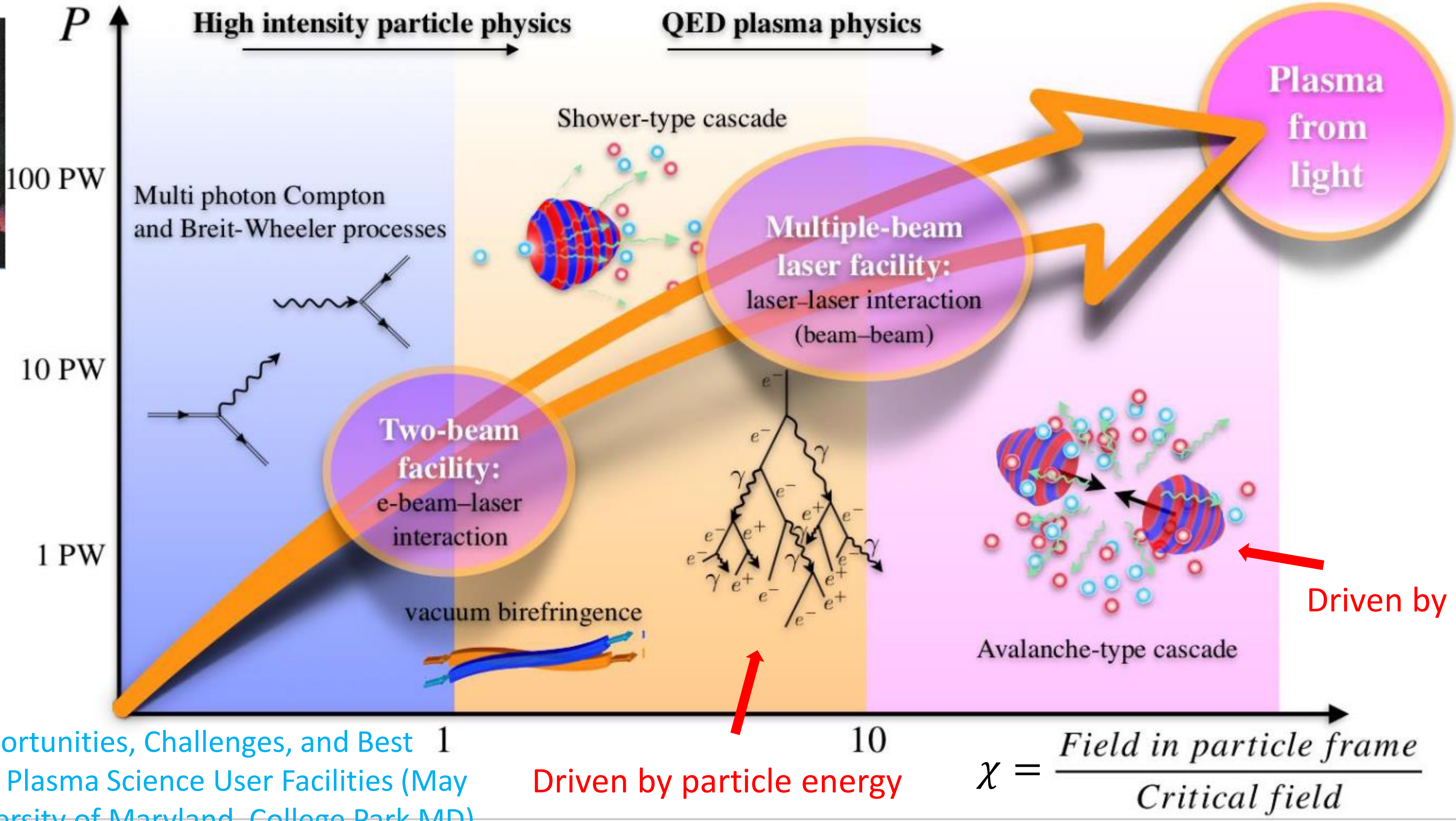
Heavy ion collisions



https://cds.cern.ch/

QED cascades (shower vs avalanche)





Driven by laser energy

Driven by particle energy

Workshop on Opportunities, Challenges, and Best Practices for Basic Plasma Science User Facilities (May 20-21, 2019, University of Maryland, College Park MD)

Schwinger pair production

At critical fields $E_{cr} \sim 10^{18}$ V/m in laboratory frame

Decay rate in constant electric field: $$\Gamma \propto \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\pi n \frac{E_{cr}}{E}\right)$$

Even stronger fields?

QED stops being perturbative in radiation fields

Mironov, Meuren, Fedotov PRD 102 (2020) 053005

$$\frac{(n+1)}{n} \sim \alpha\chi^{2/3}$$

When $\alpha\chi^{2/3} \geq 1$ Ritus-Narozhny conjecture

$\chi \geq 1600$ (remember – this means $1600\ E_{cr}$ in R.F.)

Strong-field electrodynamics in Flying focus pulses

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Stefan Weber
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and other people working on Flying Focus fields at LLE, IST, and UCLA

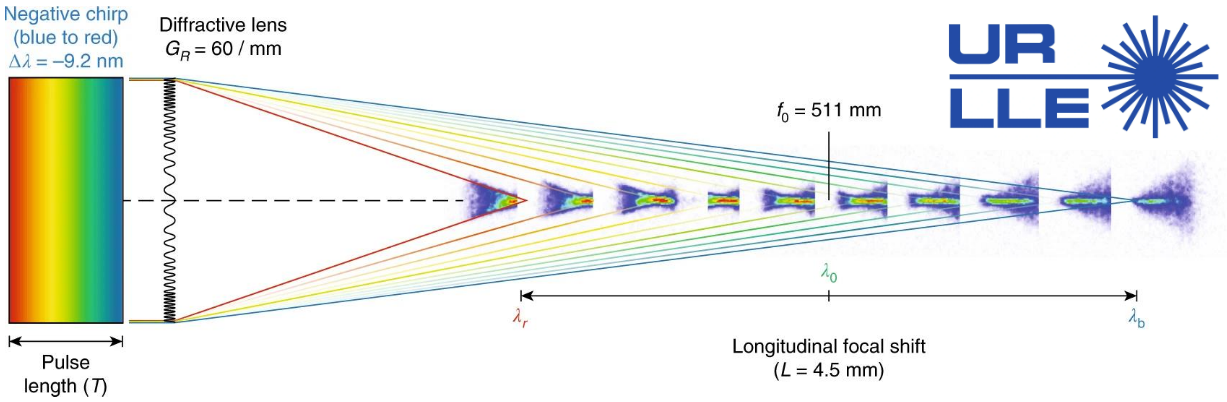
Manfred Virgil Ambat
Philip Franke
Dustin Froula
Bernardo Malaca
Warren Mori
Miguel Pardal
Jacob Pierce
Jeremy Pigeon
Hans Rinderknecht
Jessica Shaw
Tanner Simpson
David Turnbull
Jorge Vieira
Kathleen Weichman

- MF, D. Ramsey, J. P. Palastro, A. Di Piazza, Phys. Rev. A **105** (2022) L020203,
- MF, J. P. Palastro, M. Vranic, D. Ramsey, A. Di Pizza, Phys. Rev. E **107** (2023) 055213
- MF, J. P. Palastro, D. Ramsey, S. Weber, A. Di Piazza, arXiv:2307.11734

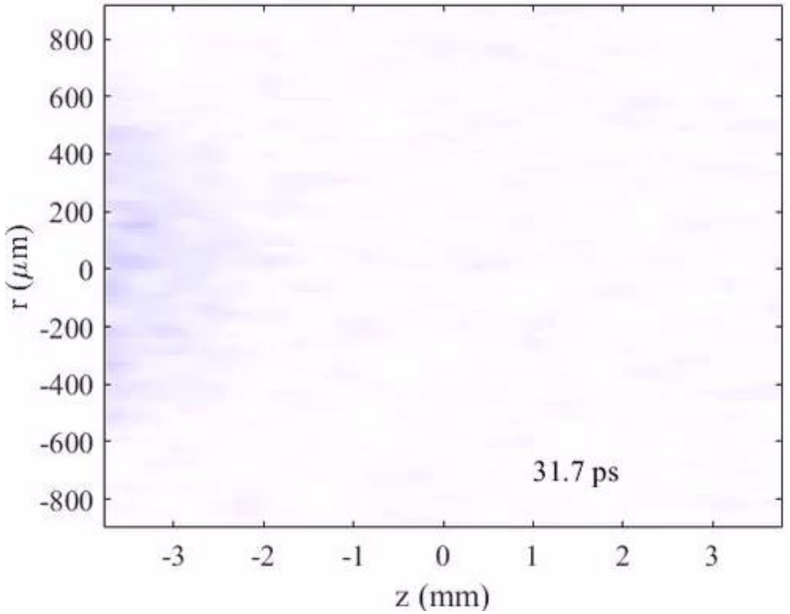
This project is supported by EU Marie Sklodowska-Curie grant No. 101105246-STEFF

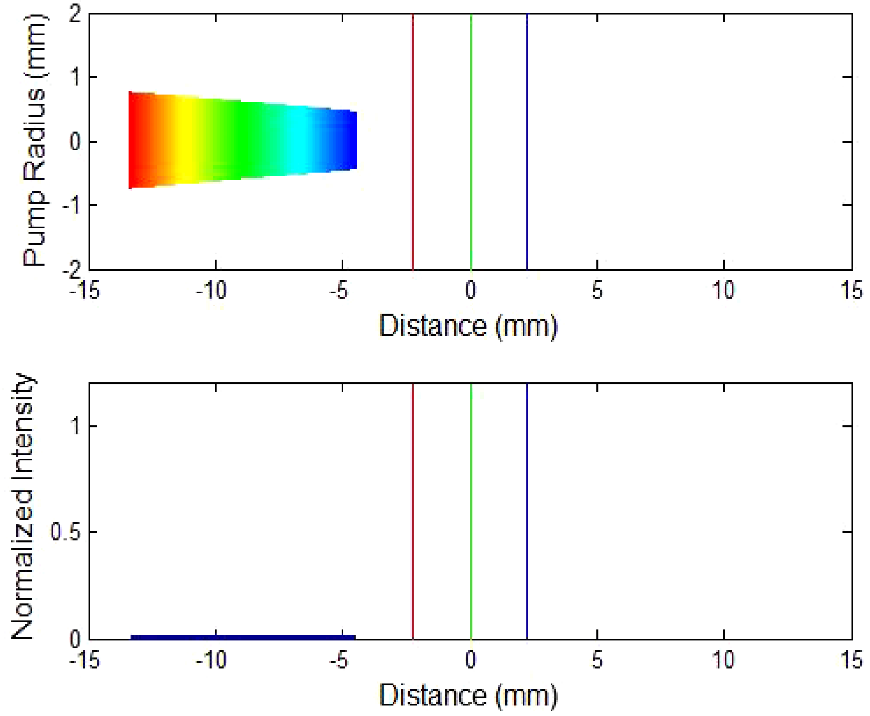
What are Flying Focus pulses? Spatiotemporal control of laser peak intensity!

a) Chromatic flying focus:

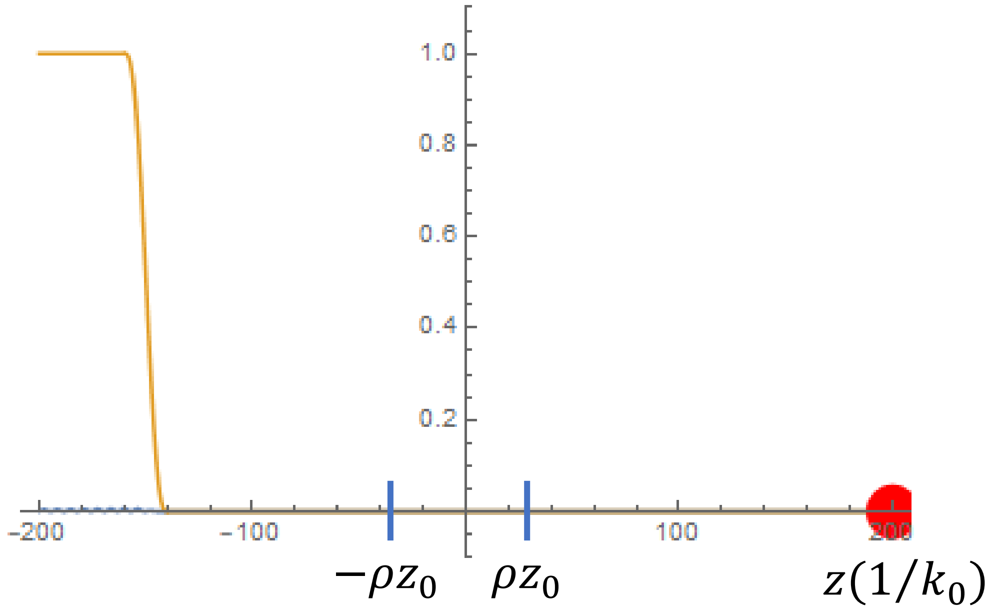


Froula et al., Nat. Photonics **12** (2018) 262.





e^- interacting with Gaussian pulse

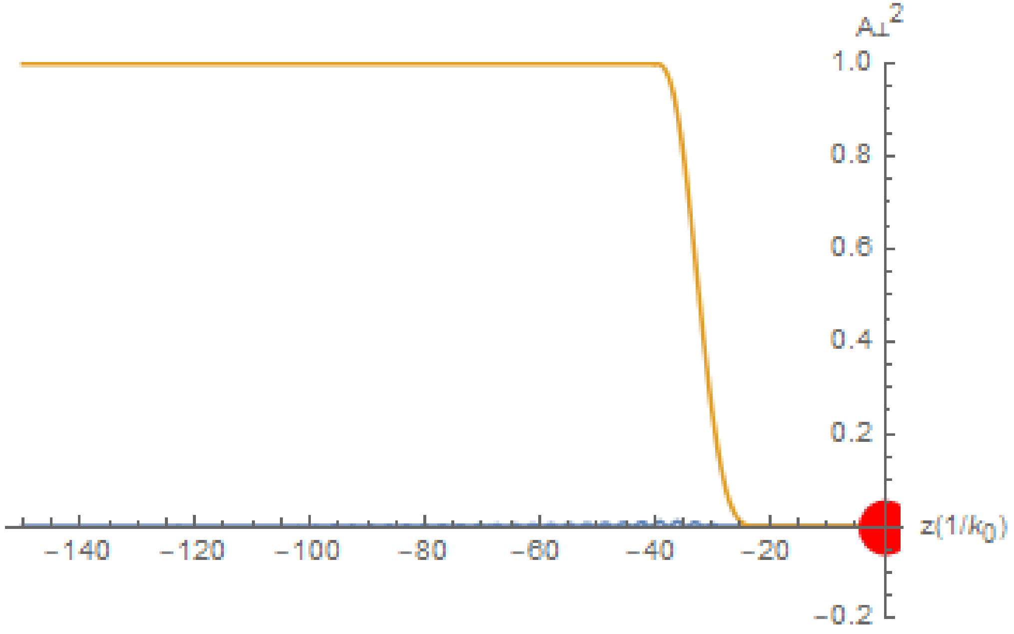


Electron on the axis interacts approximately with a field

$$\boldsymbol{A}_\perp \approx \frac{\boldsymbol{A}_0}{\sqrt{1+z^2/z_0^2}}\cos(2k_0 z)$$

ρ – Interaction length / 2 (in units of Rayleigh range)

e^- interacting with Flying focus pulse



Electron at $\eta \approx 0$ (at focus) sees approximately field:

$$\boldsymbol{A}_\perp \approx \boldsymbol{A}_0 e^{-r^2/2\sigma_0^2}\cos(2k_0 z)$$

Which is on the axis $r = 0$

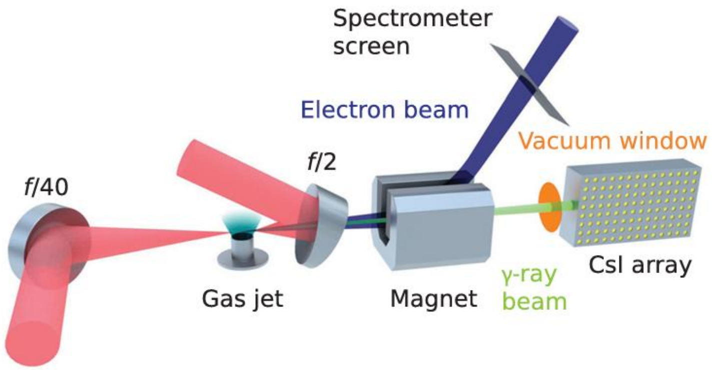
$$\boldsymbol{A}_\perp \approx \boldsymbol{A}_0\cos(2k_0 z)$$

just plane wave field!

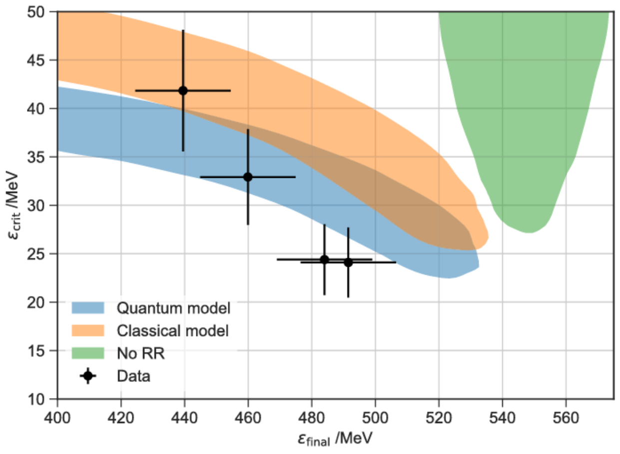
Use of high-intensity lasers for RR measurements

Cole et al., PRX 8 (2018) 011020; Poder et al., PRX 8 (2018) 031004 Astra-Gemini

Laser parameters: $E_{\mathrm{tot}} \sim 10$ J, $\xi_0 \sim 10$, $P_{\mathrm{ave}} \sim 0.25$ PW, $t_{\mathrm{int}} \sim 40$ fs



Images (top, bottom) from Cole et al., PRX 8 (2018) 011020



Deceleration formula

$$\gamma(t) = \frac{\gamma_0}{1+\kappa(t)}$$

Where the deceleration factor $\kappa(t)$ is for both ultrashort Gaussian pulses and FF pulses

$$\kappa_{\mathrm{FFP}}(t_{\mathrm{int}}) = 2.03\,\frac{E_{tot}[\mathrm{J}]\mathcal{E}_0[\mathrm{GeV}]}{\sigma_0^2[\mu\mathrm{m}]}$$

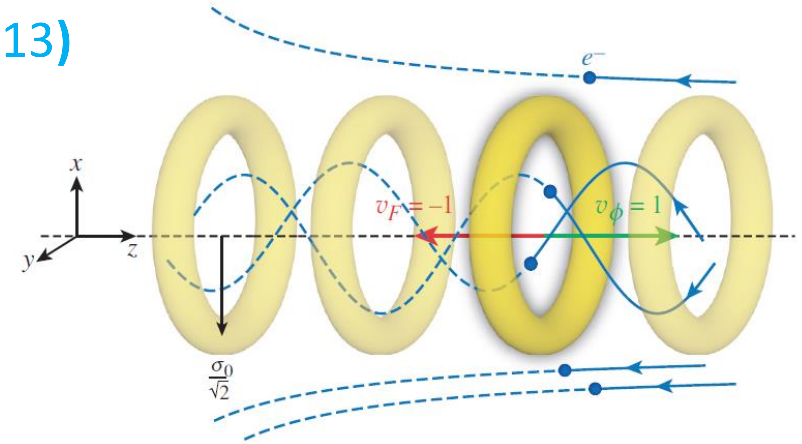
Long flying focus pulses (tens of ps) reach the same RR deceleration as ultrashort gaussian pulses (tens of fs) of the same energy

(MF et al., PRA **105** (2022) L020203)

$$P_{\mathrm{ave}} \propto a_0^2 \propto 1/t_{\mathrm{int}}$$

The necessary power is up to 10^3 times lower and ξ_0 is up to ~ 30 times lower

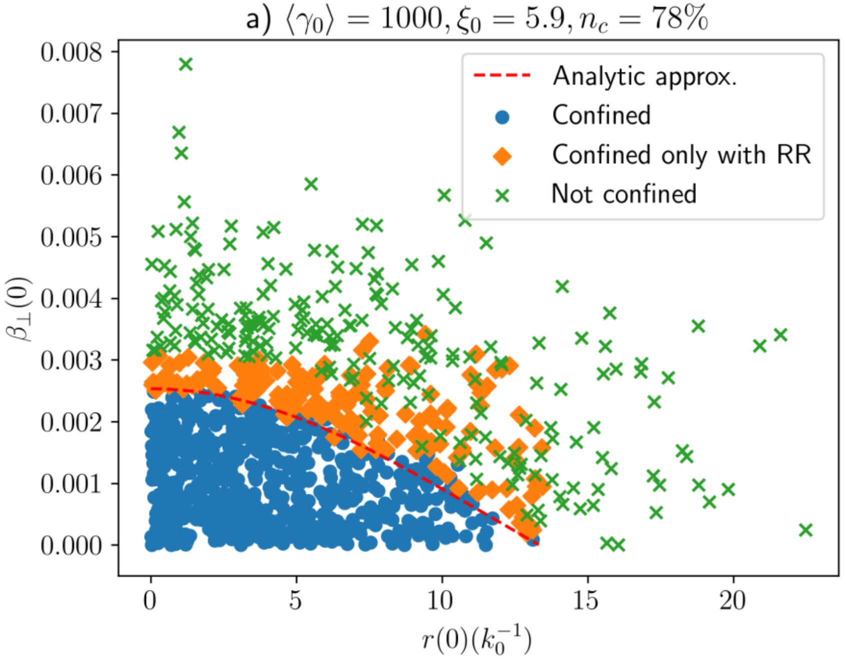
Charged particle beam control (MF et al., PRE **107** (2023) 055213)



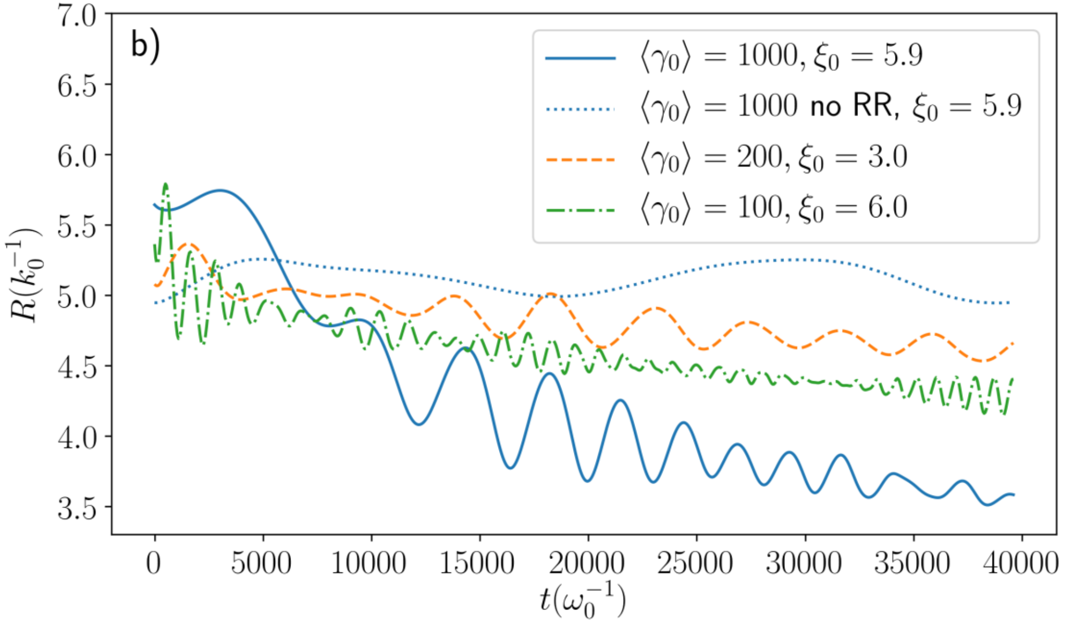
Ponderomotive potential barrier in $\ell = 1$ OAM FF pulse:

$$\overline{|\boldsymbol{A}_\perp|^2}\Big|_{\eta=0} = \frac{A_0^2 r^2}{\sigma_0^2} e^{-2r^2/\sigma_0^2}$$

which travels with the electron bunch!



Initial transverse phase space



Evolution of the RMS radius $R(t)$

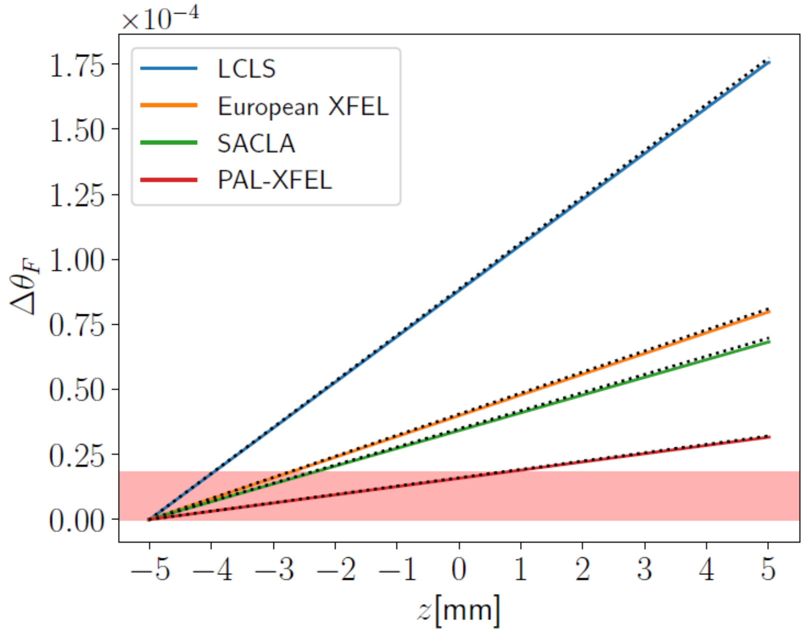
$$T = 2\pi \frac{\gamma \sigma_0}{\xi_0}$$

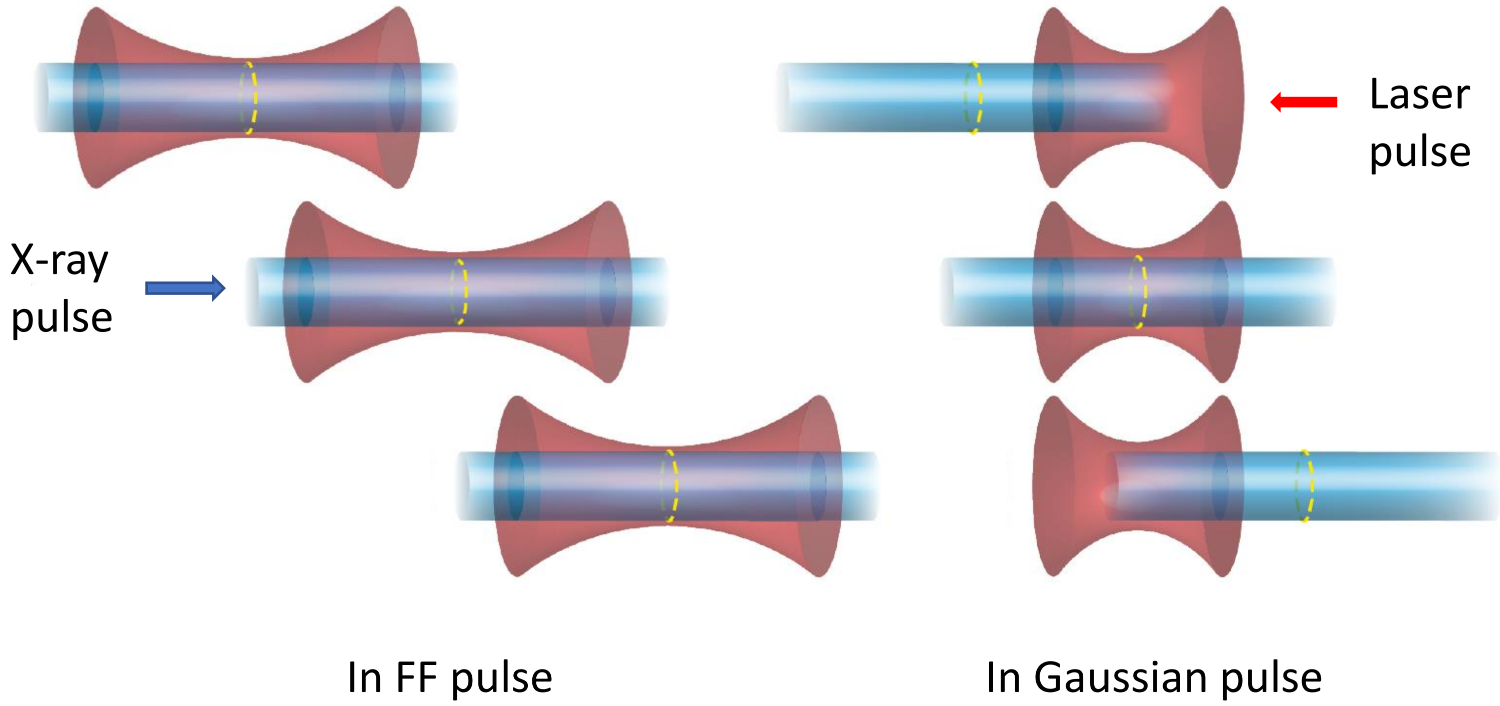
Vacuum birefringence in FF pulses (MF et al., arXiv:2307.11734)

Phase shift:

$$\Delta\theta_\ell = \frac{32\alpha^2}{15}\frac{\mathcal{E}_\ell}{e^2E_{cr}^2}\frac{\hbar c}{\lambda_\gamma\sigma_\ell^2}\Sigma_\ell\Lambda_\ell$$

Σ_ℓ - transverse overlap
Λ_ℓ - longitudinal overlap





Necessary power: $$P_F = \frac{\text{Interaction distance in FF pulse}}{\text{Interaction distance in G pulse}} P_G$$

with $\lambda_F = 1\ \mu\text{m}$, $\sigma_F = 3\ \mu\text{m}$, $P_F = 15$ TW, $\mathcal{E}_F = 1$ kJ

Thank you for your attention!

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