Investigating the locally monochromatic approximation to a two-vertex process

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Outline

1) *Trident Experiments*

2) *Model:* $\mu^- \to \mu^- e^+ e^-$

- *Two-Step subprocess*
- *One-Step subprocess*

3) *Outlook*

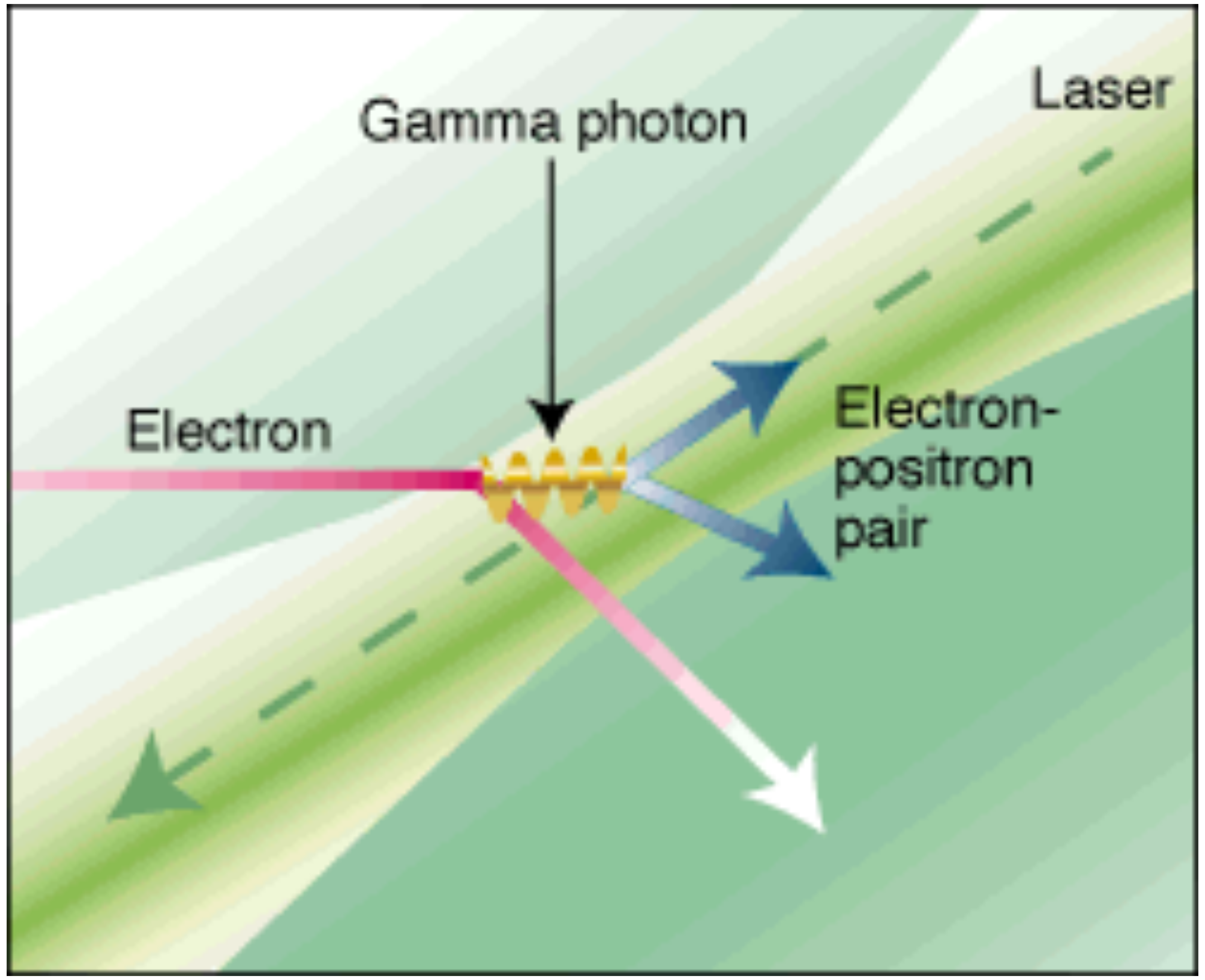
Motivation — Theory & Experiments

Why revisit one-step vs two-step?

SLAC E144 — multiphoton QED test

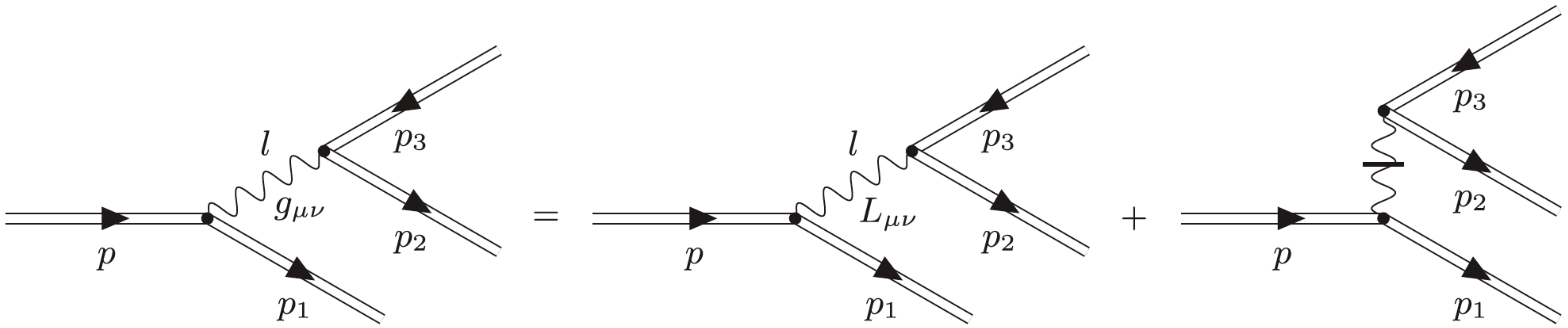
- 46.6 GeV electrons on intensive laser
- Perturbative regime (Field-strength : $\chi_0 \sim 0.3$, Intensity: $\xi = a_0 \sim 0.3$)
- Observed multiphoton Compton scattering + multiphoton Breit–Wheeler

PRL. 76, p. 3116 (1996)



https://www.slac.stanford.edu/exp/e144/

Two-Step and One-Step



- Two-Step:
  - Factorises into sequential NLC × NBW subprocesses.
  - Photon propagator: both on-shell pole **and** off-shell part contribute.
  - Off-shell part generates causality-preserving Heaviside function that enforces time ordering.
- One-Step:
  - Non-factorisable part of the full diagram.
  - Involves virtual photon exchange across both vertices.
  - Cannot be captured by **cascade approximation** (n-Step).

Motivation — Theory & Experiments

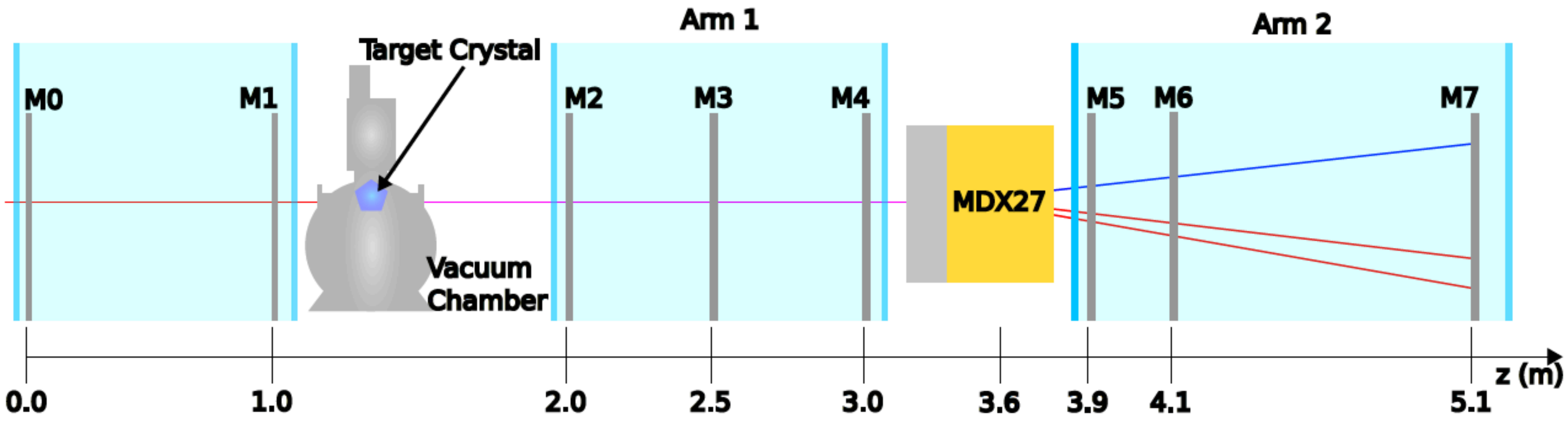
CERN NA63:

PRD,108.052013 (2023)

PRL, 130, 071601 (2023)

Differential study:
Mostly two-step.
But hints of one-step

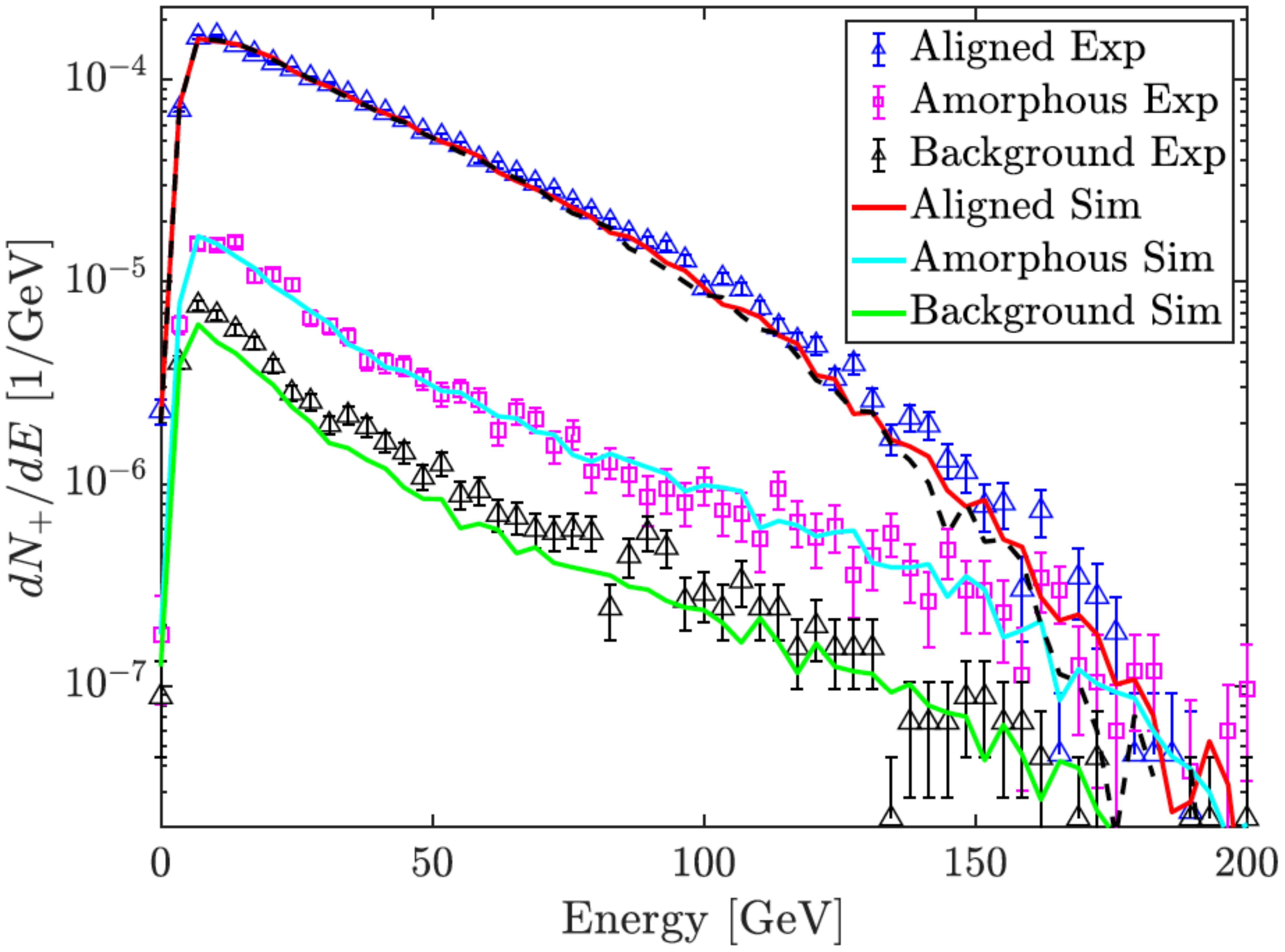
- Beam energy: 200 GeV electrons
- Target: 400 μm germanium crystal, ⟨110⟩ axis
- Strong-field parameter: $\chi_0 \lesssim 2.4$
- Intensity parameter: $\xi \approx 18$



Motivation — Theory & Experiments

CERN NA63 — Key Results

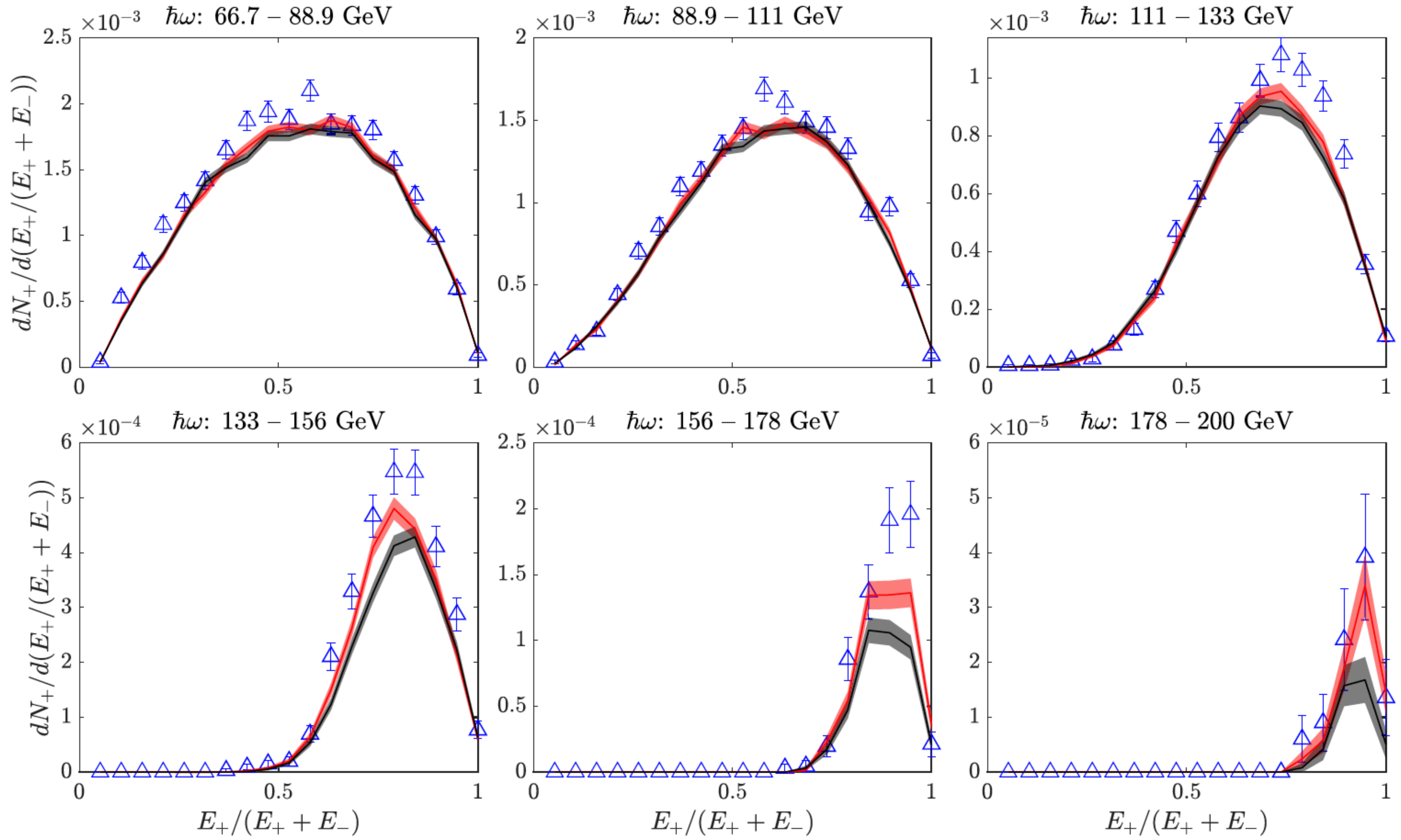
- Observed electron trident: $e^- \rightarrow e^- e^+ e^-$
- Precision measurement: Overall yields match the local constant field approximation (LCFA) based on the two-step calculation.



PRL, 130, 071601 (2023)

Motivation — Theory & Experiments

- ○ Differential spectra: saw indications of one-step contribution in high-energy tail



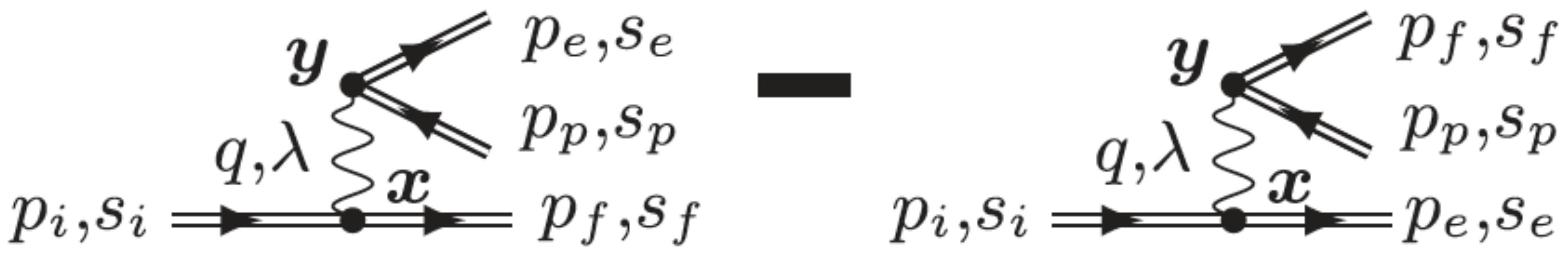
- Triangles: Experimental data
- Red line: One-Step+ Two-Step
- Black line: Only Two-Step

Real motivation for revisiting the one-/two-step separation.

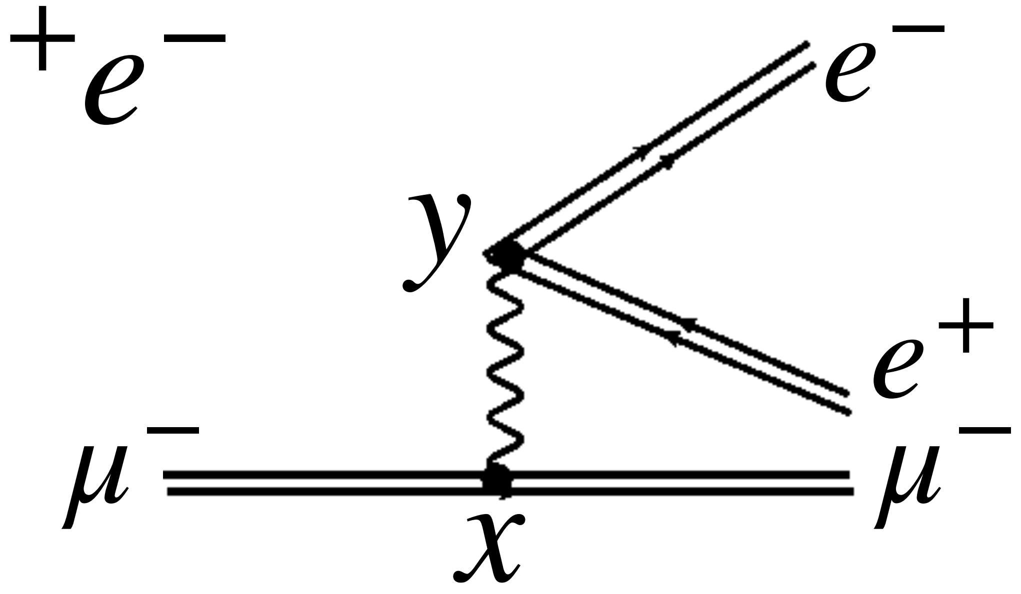
Muon Trident Process – Clean Test Case

Muon probe avoids crossing diagrams, heavier mass gives scale separation

$$e^- \to e^- e^+ e^-$$



$$\mu^- \to \mu^- e^+ e^-$$



- Allows cleaner theoretical study of one-step vs two-step

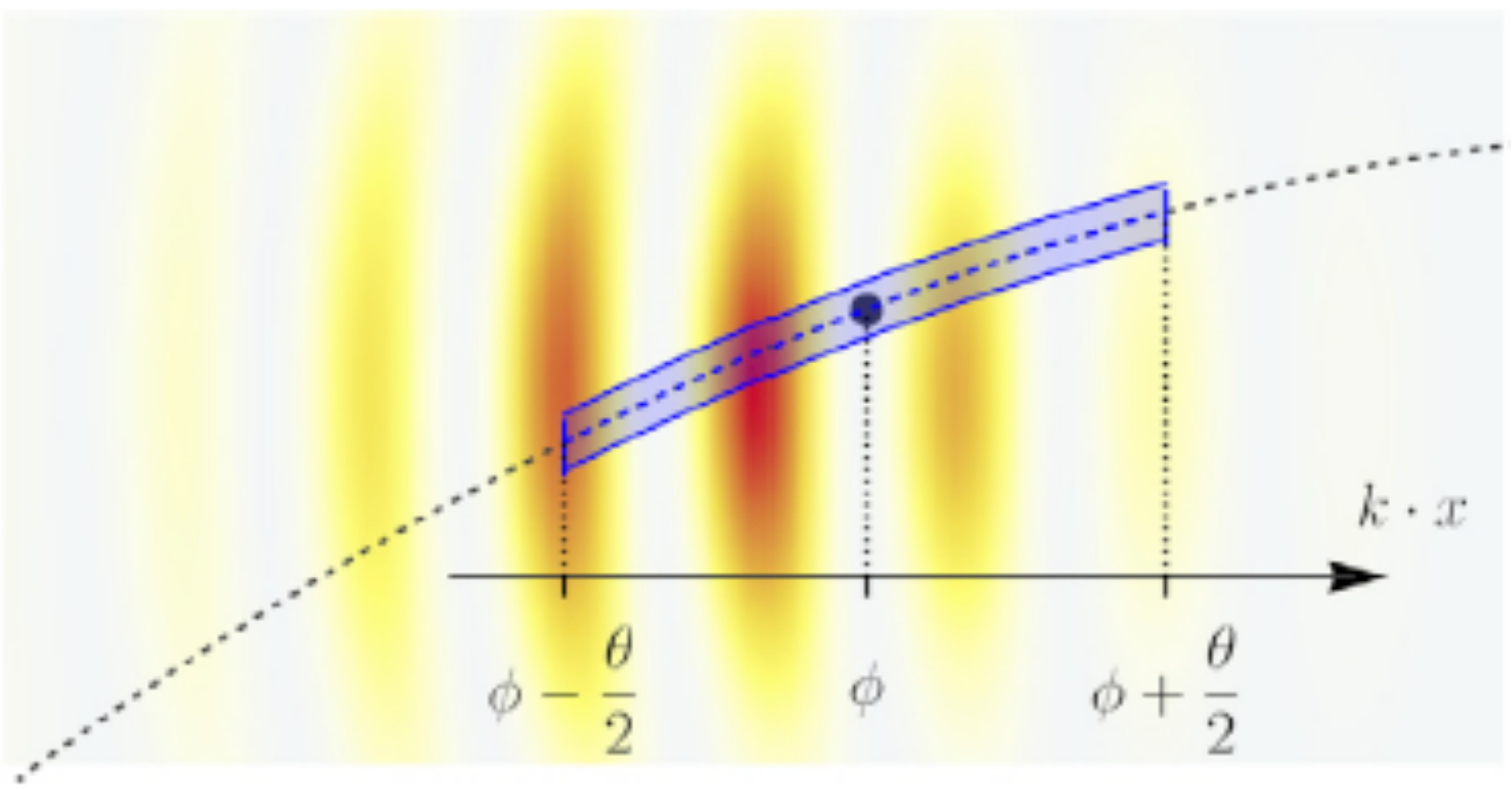
- Recent **all-optical experiments** have produced muons → future relevance:

  2024 (Zhang et al. Nat. Phys. 21,1050, 2025**)**: First proof-of-principle muon generation with ultrashort high-intensity laser → GeV e^- on Pb converter; muons confirmed via lifetime measurements.
  2025 (Calvin et al. arXiv:2503.20904**)**: Directional muons produced at PW-LWFA → 95% detection confidence.

- Muon collider and new physics.

Framework: Locally Monochromatic Approximation (LMA)

- **Adiabatic approximation**: slow envelope local, fast oscillations exact



$$|\mathscr{M}|^2 = \int d\varphi d\varphi' M(\varphi) M^*(\varphi')$$

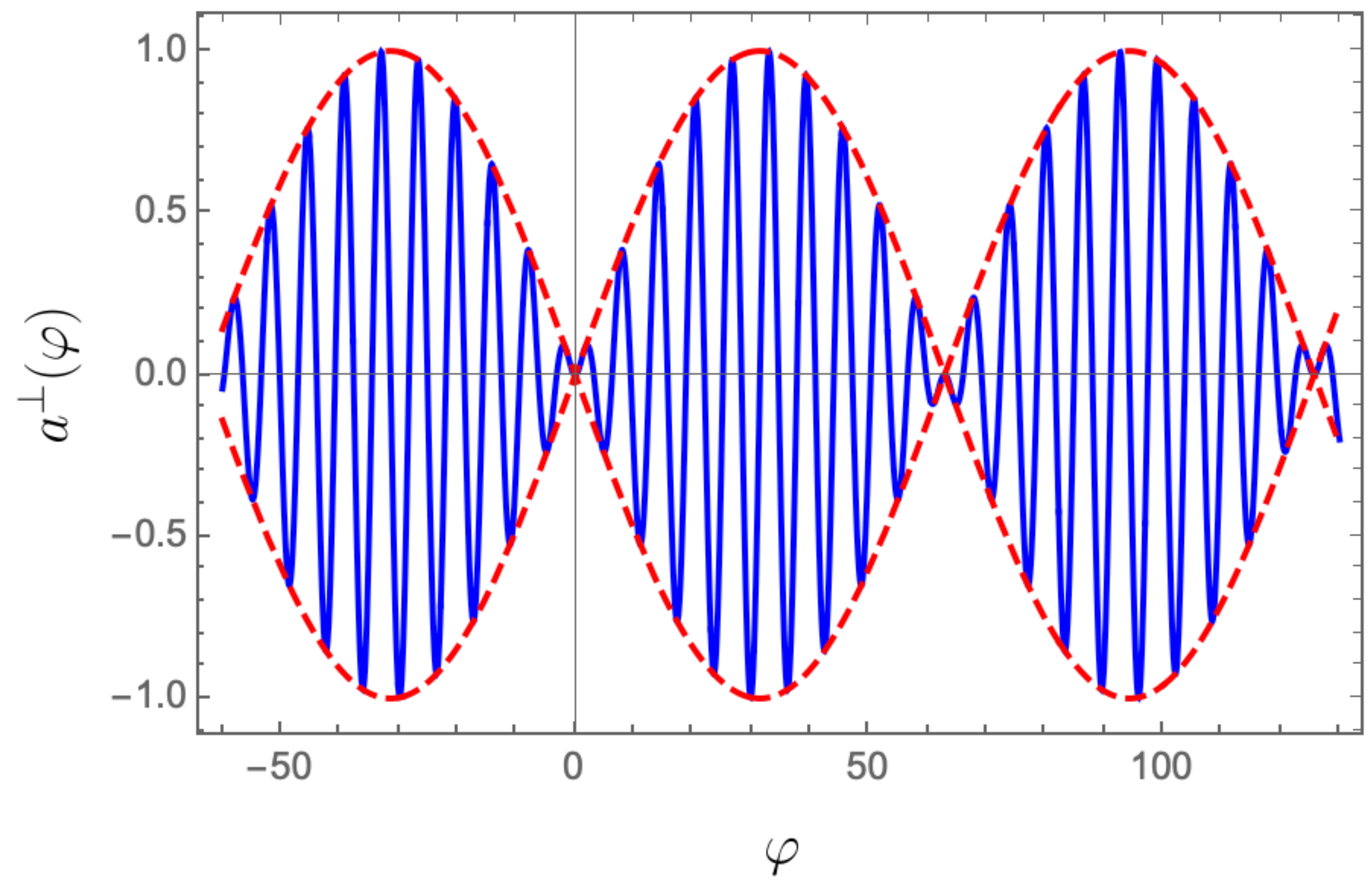
$$\phi = \frac{\varphi + \varphi'}{2}, \quad \theta = \varphi - \varphi'$$

PRA, 102, 063110 (2020);
Phys. Rep. 1010 (2023) 1–138.

$$a^{\perp}(\varphi) = m\xi P(\varphi)(\cos\varphi, \sin\varphi)$$

Slow varying profile

$$P(\varphi') \approx P(\varphi) \approx P(\phi)$$



Framework: Locally Monochromatic Approximation (LMA)

$$M = \mathscr{A} \exp\{i\,.\,\text{oscillating} + i\,.\,\text{const}\}$$

- **Harmonics structure**, provides correct in infrared limit

Jacobi-Anger expansion

$$e^{-iz\sin\varphi} = \sum_{n=-\infty}^{\infty} J_n(z) e^{-in\varphi}$$

Decompose into plan-waves

$$M = \sum_n M_n$$

LMA is already used in interpreting E144 data

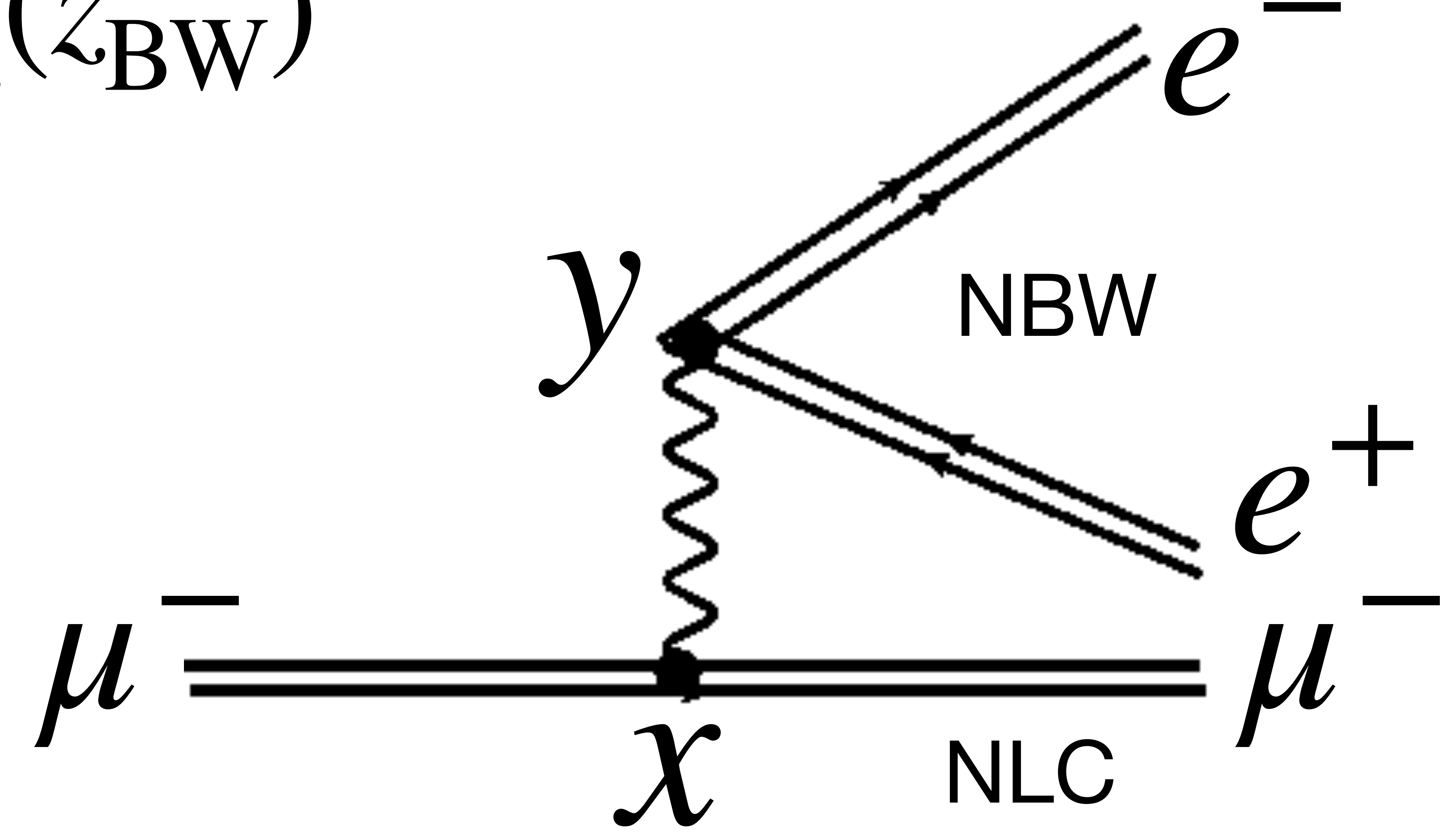
Method & Key Idea

- Use **LMA** to expand μ-trident amplitude and separate contributions.
- Study how **pulse length, frequency, strength** affect the balance between one-step and two-step terms.
- **Question:** For given parameters and observables, how accurate is the cascade (two-step) approximation?

LMA for μ-Trident

$$\frac{d^2P_{\text{tri}}}{ds_e ds_\mu} \sim \sum_{\{n_i\}} C_{n'_x n_x n'_y n_y} J_{n'_x}(z_C) J_{n_x}(z_C) J_{n'_y}(z_{\text{BW}}) J_{n_y}(z_{\text{BW}})$$

$$n'_i \in \{n_i, n_i \pm 1\}$$



n_y: net photons absorption/emission from electron

n_x: net photons absorption/emission from muon

s_μ, s_e are scattered muon and positron light fractions respectively.

LMA for μ-Trident

Structure of coefficients:

$$C_{n'_x n_x n'_y n_y} \propto \delta(n_x + n_y - n_{x,*} - n_{y,*}) \frac{\sin\left[2(\phi_y - \phi_x)(n_y - n_{y,*})\right]}{n_y - n_{y,*}}$$

Threshold harmonics orders:

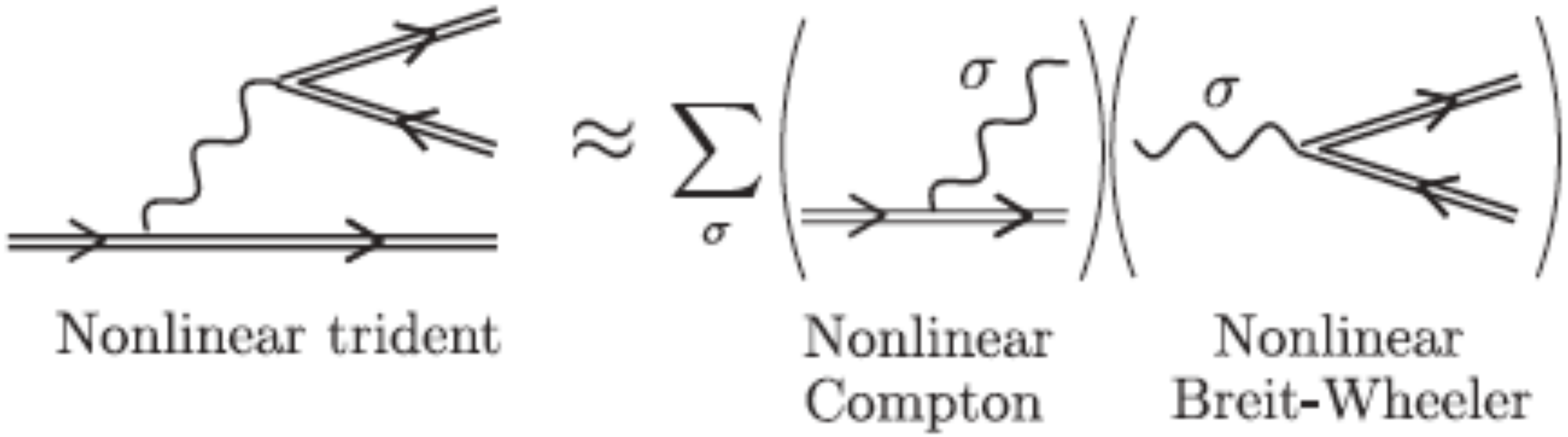
$$n_{x,*} = A_C Q_C^{\perp 2} + B_C \qquad n_{y,*} = A_{\mathrm{BW}} Q_{\mathrm{BW}}^{\perp 2} + B_{\mathrm{BW}}$$

Constraints on momentum variables

$$Q_C^{\perp 2} = \frac{1}{A_C}\left(n_x + n_y - A_{\mathrm{BW}} Q_{\mathrm{BW}}^{\perp 2} - B_C - B_{\mathrm{BW}}\right) > 0$$

Two-Step Limit

Two-step valid: Long pulse, $\phi_i \gg 1$, and non-linear regime, $\xi \gg 1$.



$\sigma = ||, \perp$ is the on-shell photon polarisation

For large separation: $\phi_y \gg \phi_x$ → $\lim_{a\to 0} \frac{1}{a}\mathrm{sinc}\left(\frac{x}{a}\right) \to \delta(x)$

$$C_{n_x' n_x n_y' n_y} \propto \delta(n_x + n_y - n_{x,*} - n_{y,*}) \frac{\sin\left[2(\phi_y - \phi_x)(n_y - n_{y,*})\right]}{n_y - n_{y,*}}$$

→ $$C_{n_x' n_x n_y' n_y} \propto \delta(n_x - n_{x,*})\delta(n_y - n_{y,*})$$

→ $$Q_C^{\perp 2} = \frac{n_x - B_C}{A_C} \qquad Q_{\mathrm{BW}}^{\perp 2} = \frac{n_y - B_{\mathrm{BW}}}{A_{\mathrm{BW}}}$$

Results: Two-Step Case

Replace the metric tensor into polarisation sum:

- $g_{\mu\nu} \rightarrow h_{\mu\nu} = -\varepsilon_{\mu}^{\parallel}\varepsilon_{\nu}^{\parallel} - \varepsilon_{\mu}^{\perp}\varepsilon_{\nu}^{\perp}$

Long pulse, for non-zero integers, m_i:

- $\langle e^{im_x\phi_x}\rangle \rightarrow 0 \,, \langle e^{im_y\phi_y}\rangle \rightarrow 0$

We find that

$$\frac{d^2P_{\text{tri}}}{ds_\mu ds_e} = \frac{dP_C^{\perp}}{ds_\mu}\frac{dP_{\text{BW}}^{\perp}}{ds_e} + \frac{dP_C^{||}}{ds_\mu}\frac{dP_{\text{BW}}^{||}}{ds_e}$$

One-Step Case

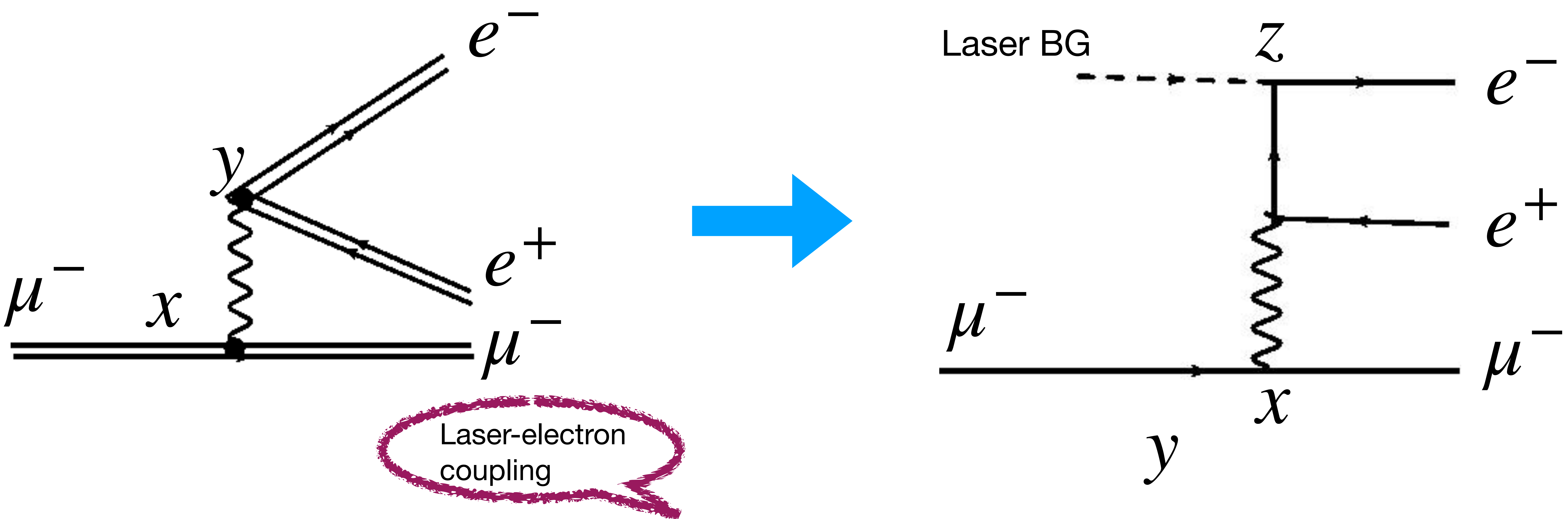
One-step cannot be neglected: shorter pulses or higher frequencies.

$$\Rightarrow \quad |z| \ll 1$$

$$J_n(z) \sim \frac{1}{n!}\left(\frac{z}{2}\right)^n \quad \Rightarrow$$

Higher orders harmonics contributions are exponentially suppressed

One-Step Limit



Expanding LMA expression through ξ_e;

laser-muon coupling: $\xi_\mu \to 0$

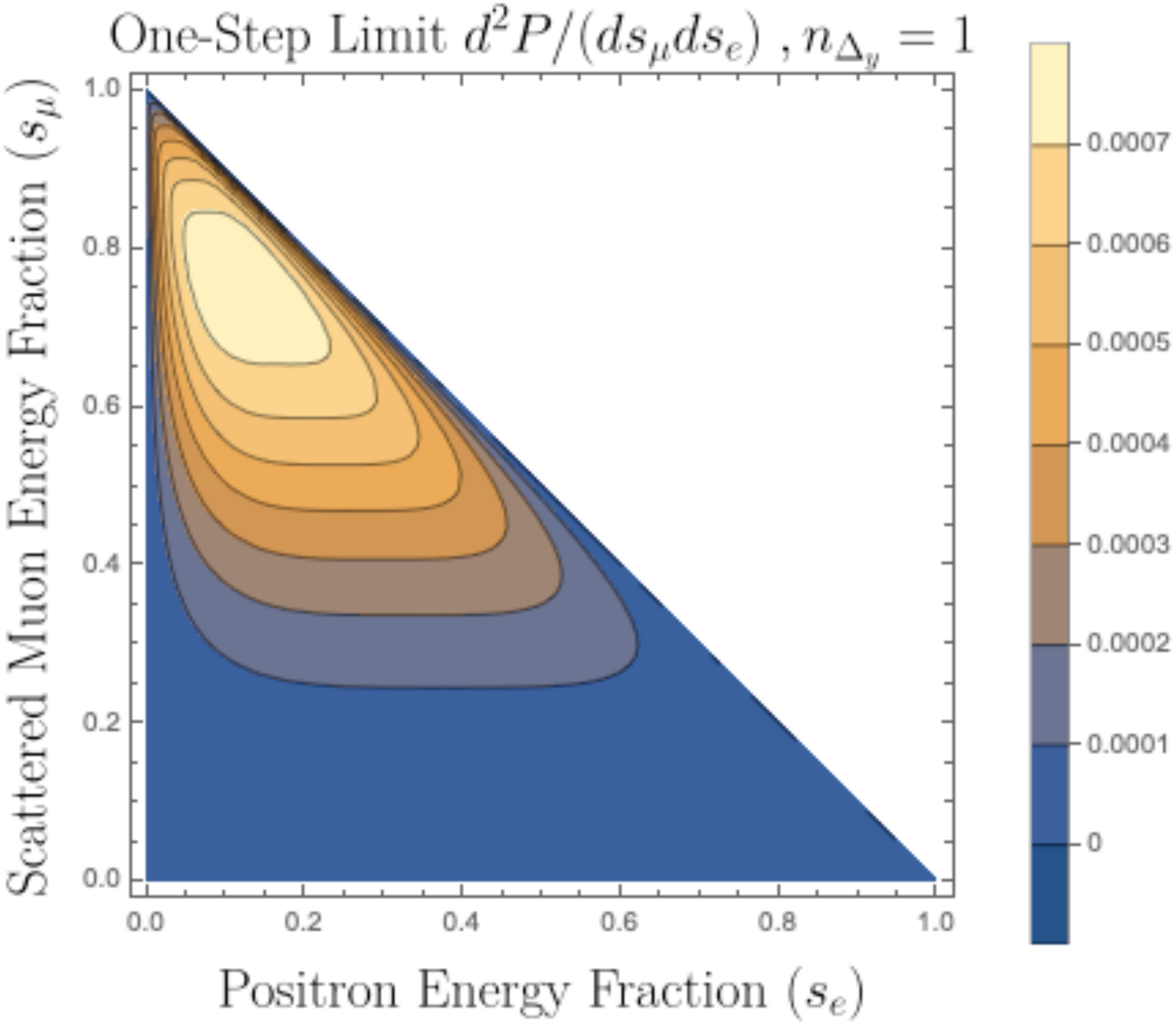
$$\bar{u}_e \not{a} \frac{i}{\not{k} - m_e - i\delta} \gamma^\mu v_e D_{\mu\nu} \bar{u}_\mu \gamma^\nu u_\mu$$

Leading order contribution: $n_x = 0$, $n_y = 1$

Match to one-step Feynman diagram

$$P^{\mathrm{LMA}} \to P^{\mathrm{Feyn}}$$

Example : One-Step Contributions



- LMA agrees with Feynman
- Higher harmonics are negligible

Work in progress: Inclusion in numerical calculations efficiently

Outlook

- What we want to calculate with LMA:
  - NA63 spectra → reproduce their claim
  - Quantify accuracy of two-step approx. under given parameters
  - Identify regime where one-step dominates
- Future: apply to muon–laser and electron–laser experiments.

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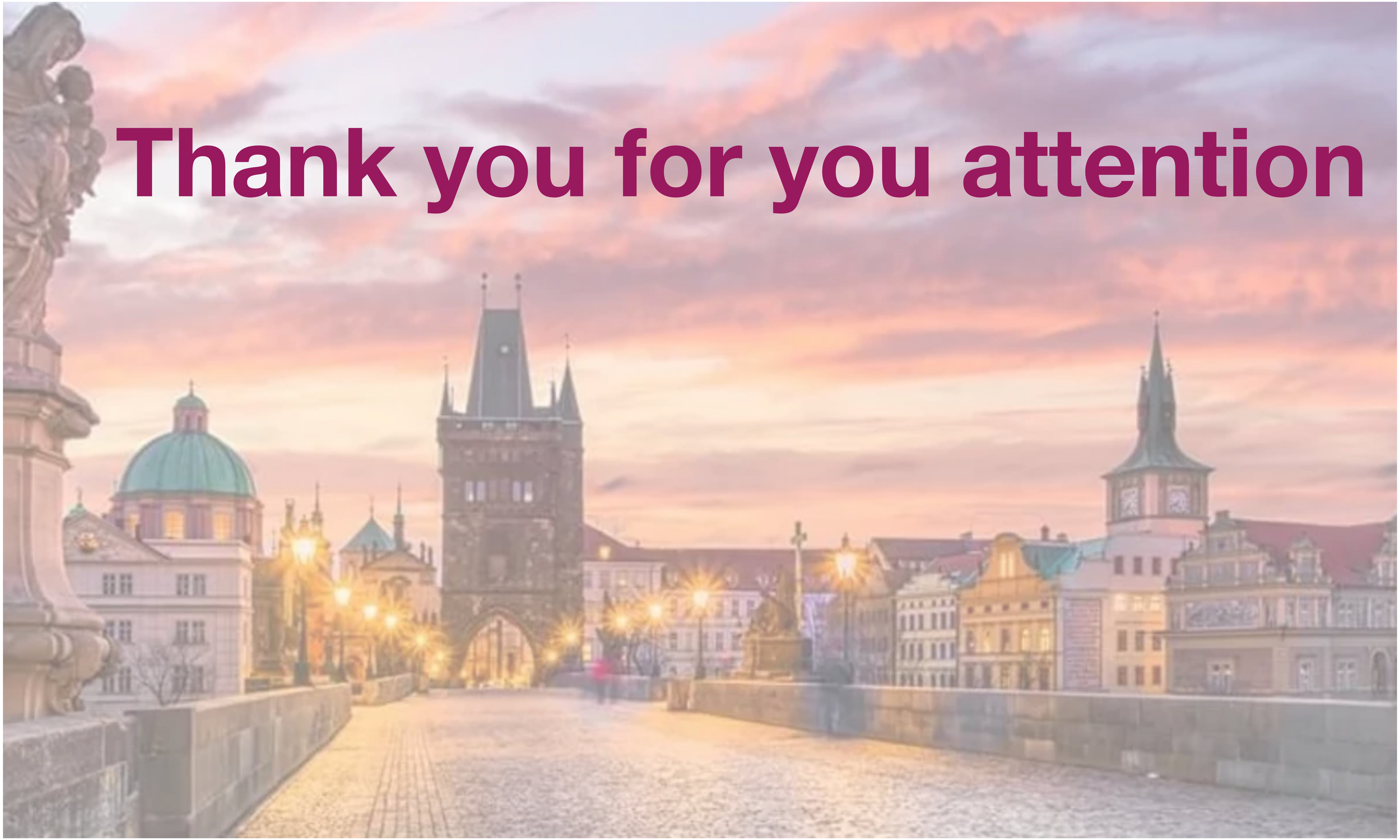
Thank you for you attention



Backup

Cascade Approximation — When Does It Work?

- **Cascade approximation:**
  n^{th} order process ≈ sequence of 1^{st}-order subprocesses (e.g. NLC, NBW).

- **Works well** in long-pulse / LCFA limit → NA63 total yields matched.

- **But:** NA63 spectra show discrepancies → hints of **virtual (one-step) pathways**.

- **Key question:** When is the cascade picture accurate, and when must full 2-vertex description be included?

- Long-pulse → sequential subprocesses
- NA63 suggests need to account for **one-step**

1. Two key parameters: a_0 and χ_e

- $a_0 = \frac{eE}{m\omega}$: **classical nonlinearity parameter** → how strong the field is in units of the electron's rest energy over one laser cycle.
    - $a_0 \gtrsim 1$: electron quiver motion is relativistic → nonlinear effects (multiple laser photons can be absorbed).
- $\chi_e = \gamma \frac{E}{E_S}$: **quantum nonlinearity parameter** → whether quantum recoil & pair production are relevant.
    - $\chi_e \ll 1$: pair production suppressed, radiation mostly classical.
    - $\chi_e \sim 1$: quantum effects (stochastic emission, pair creation).

So:

- Nonlinear QED ≠ requires large χ_e.
- Nonlinear behaviour already appears if $a_0 \gtrsim 1$, even with $\chi_e < 1$.

- **2024 (Terzani et al.)**: BELLA PW laser produced directional, high-flux muon beams—orders-of-magnitude above cosmic rays.

- **Simulations (2023–2025)**: Proposed systems (e.g., ELI-NP and GIST) predict muon yields up to 10^4–10^7 p shot—suitable for applications like muon radiography and spectroscopy.

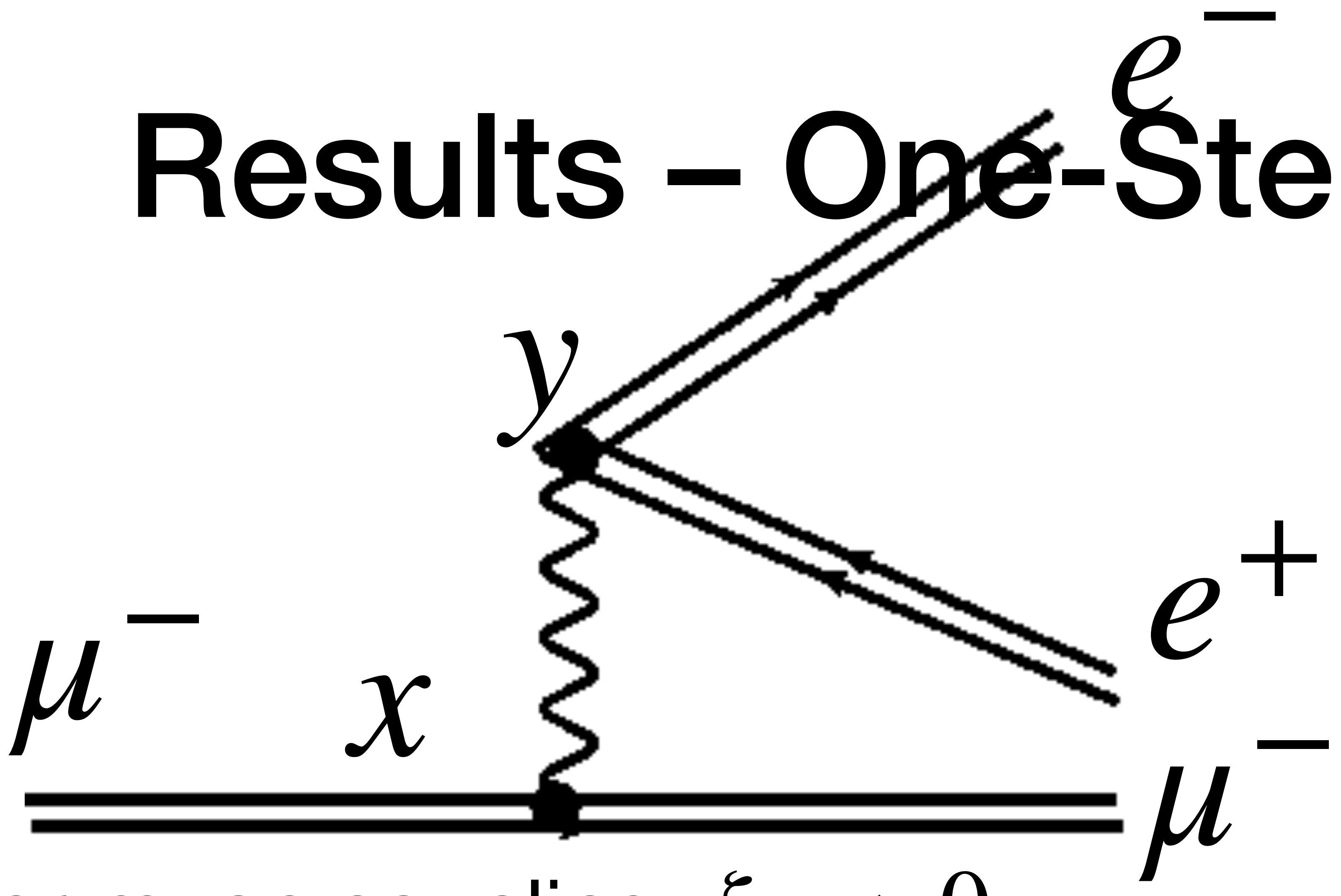
$|z| \ll 1$

$$J_n(z) \sim \frac{1}{n!}\left(\frac{z}{2}\right)^n$$ → Higher orders are exponentially suppressed

One-step cannot be neglected: shorter pulses or higher frequencies.

Match to one-step Feynman diagram $P^{\mathrm{LMA}} \to P^{\mathrm{Feyn}}$

Results – One-Step Contributions



Laser-muon coupling: $\xi_\mu \to 0$

$n_x = 0\ , n_y = 1,$

$J_0\left(Z_C\right) \to 1\ , \quad J_1\left(Z_{\mathrm{BW}}\right) \to \frac{Z_{\mathrm{BW}}}{2}$

→