The inadequacy of the established analytical ion recombination models for ionization chamber dosimetry at conventional dose rates

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Volume recombination theory in conventional dose per pulse

The code of practice for the determination of the absorbed dose to water in external beam radiotherapy recommends the use of the two voltage method for the determination of the ion recombination correction factor (k_s). They are based on Boag's early theory that do not account for free electrons drifting inside the volume.

According to Boag's theory the saturation factor for pulsed beams can be calculated as[1]:

$$k_s = \frac{u}{\log(1+u)}$$

where u is a dimensionless parameter that depends on the ionization chamber (IC) geometry and dimensions, operating bias voltage and several physical constants.

It can be shown that two collected charges (Q_1, Q_2) at two operational bias voltages (V_1, V_2) where $V_1 > V_2$ are related by the following expression:

$$\frac{Q_1}{Q_2} = \frac{V_1}{V_2}\frac{\log(1+u)}{\log(1+uV_1/V_2)}$$

This transcendental equation can be solved for u and no information about any physical constant or IC dimensions is needed.

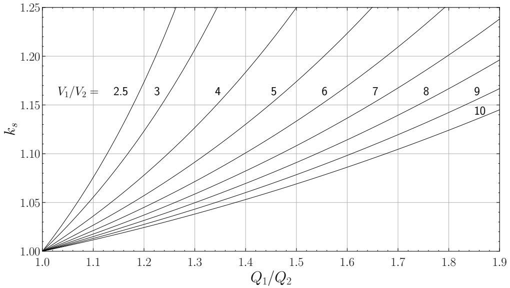


Figure: k_s as a function of voltage quotient and charge quotient. Figure reproduced from Boag[2]

[1] Boag, J.W. *Br. J. Radiol.* **23**(274) 601-11 (1950).

A look into the TRS-398

The TRS-398 use a second-order polynomial fit to the solution of the transcendental equation based on the work of Weinhous and Meli[3]:

$$k_s = a_0 + a_1\left(\frac{Q_1}{Q_2}\right) + a_2\left(\frac{Q_1}{Q_2}\right)^2$$

Where the coefficients are tabulated[4]:

V_1/V_2	a_0	a_1	a_2
2.0	2.337	-3.636	2.299
2.5	1.474	-1.587	1.114
3.0	1.198	-0.875	0.677
3.5	1.08	-0.542	0.463
4.0	1.022	-0.363	0.341
5.0	0.975	-0.188	0.214

Table: Table of the coefficients given by the TRS-398 for the calculation of the ion recombination correction factor.

In the case that $k_s < 1.03$ the folowing aproximation is given:

$$k_s - 1 \approx \frac{Q_1/Q_2 - 1}{V_1/V_2 - 1}$$

This methods provide a calculation that is:

- Independent of the ionization chamber dimensions.
- Independent of the pulse duration (as long as the no overlapping pulses condition is fulfilled).
- Independent of the physical constants involved in the problem.

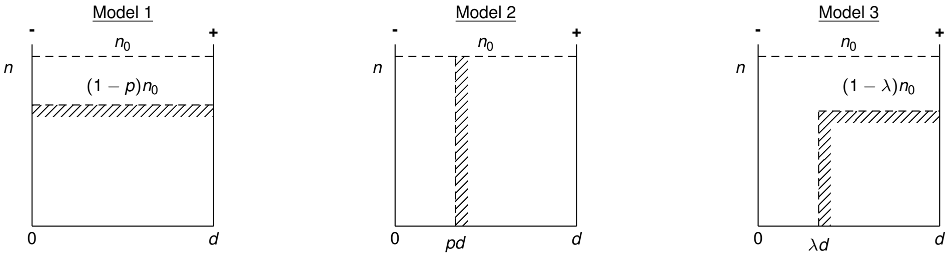
[2] Boag, J.W. (1987). Ionization chambers. Kase, K R, Bjarngard, B E, & Attix, F H. *The dosimetry of ionizing radiation. Volume 2*. United States.

[3] Weinhous, M. S. and Meli, J. A. Med. Phys. 11, 846 (1984).

[4] IAEA TRS 398. Absorbed Dose Determination in External Beam Radiotherapy. An International Code of Practice for Dosimetry Based on Standards of Absorbed Dose to Water. Tehnical Report no 398.

Boag's models with free electrons (I)

Later, Boag and contemporaries developed three models that account for the free electron fraction[5]. Each model uses a different approximation of the free electron density along the ionization chamber.



$f' = \frac{1}{u}\log\left(1 + \frac{e^{pu}-1}{p}\right)$ $f'' = p + \frac{1}{u}\log\left(1 + (1-p)u\right)$ $f'' = \lambda + \frac{1}{u}\log\left(1 + \frac{e^{\lambda(1-\lambda)u}-1}{\lambda}\right)$

Figure: Comparison of the different negative densities across the ionization chamber gap used by Boag *et al.* for the derivation of the three models.

where p is the free electron fraction and can be calculated knowing the electron velocity v_e in m s^{-1}, attachment time τ in s and ionization chamber gap d in m as:

$$p = \frac{v_e\tau}{d}\left(1 - \exp\left(-\frac{d}{v_e\tau}\right)\right)$$

the parameter λ is defined as $\lambda = 1 - \sqrt{1-p}$.

5 Boag, J.W. *et al.* *Phys. Med. Biol.* **41**(5) 885 (1996).

Boag's models with free electrons (II)

This three models present inconsistencies even at very low dose per pulse. If we take the derivative in the limit of zero dose per pulse is:

$$\lim_{D_0 \to 0} \frac{df}{dD_0} \neq \lim_{D_0 \to 0} \frac{df'}{dD_0} \neq \lim_{D_0 \to 0} \frac{df''}{dD_0} \neq \lim_{D_0 \to 0} \frac{df'''}{dD_0} = -\frac{2\,(\mu_+ + \mu_-)\ V\ W_{air}}{(1-\lambda)^3\ \alpha\ \rho_{air}\ d^2} \tag{1}$$

Symbol	Units	Definition
α	$m^3\ s^{-1}$	Volume recombination coefficient between positive and negative ions
μ_+, μ_-	$m^2\ V^{-1}\ s^{-1}$	Mobility of positive and negative ions, respectively
W_{air}	eV	Average energy per ion pair

Table: Definition of the symbols used in the equation 1.

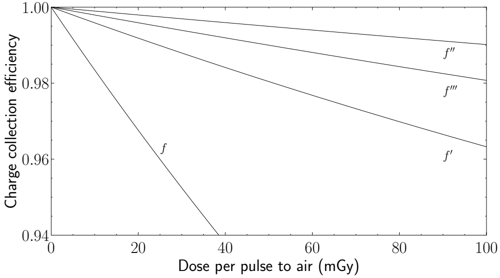


Figure: Prediction of the different models for a 1 mm IC operated at 300 V in standard temperature, pressure and humidity conditions (20 ºC, 1013.25 hPa and 50 %).

Fenwick's model

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PAPER

Collection efficiencies of ionization chambers in pulsed radiation beams: an exact solution of an ion recombination model including free electron effects

John D Fenwick[1,*] and Sudhir Kumar[2]

This inconsistencies has been recently solver by the work of Fenwick and Kumar[6] work. They develop a equation using the exact distribution of free electron along the IC:

$$f_{\text{Fenwick}} = \frac{1}{u} \log\left(1 + R\exp(R)[E_1(R\exp(-\tau d/v_e)) - E_1(R)]\right)$$

where $R = uv_e\tau/d$ and E_1 is the exponential integral function defined as $E_1(\xi) = \int_\xi^\infty \frac{\exp(-\eta)}{\eta} d\eta$

This formula **do not account** for the electric field perturbation neither the pulse duration or pulse structure.

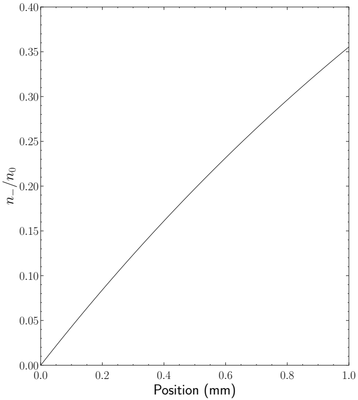

Figure: Remaining negative ion density for a 1 mm IC operated at 400 V.

Numerical model

The equations describing the charge transport inside an IC can be written as[7]:

$$
\begin{aligned}
\frac{\partial n_+(z,t)}{\partial t} &= I(z,t) - \alpha\, n_+(z,t)\, n_-(z,t) \\
&\quad - \frac{\partial}{\partial z}\left[E(z,t)\, \mu_+\, n_+(z,t)\right], \\
\frac{\partial n_-(z,t)}{\partial t} &= \gamma\, n_e(z,t) - \alpha\, n_+(z,t)\, n_-(z,t) \\
&\quad + \frac{\partial}{\partial z}\left[E(z,t)\, \mu_-\, n_-(z,t)\right], \\
\frac{\partial n_e(z,t)}{\partial t} &= I(z,t) - \gamma\, n_e(z,t) \\
&\quad + \frac{\partial}{\partial z}\left[v_e(z,t)\, n_e(z,t)\right],
\end{aligned}
\tag{2}
$$

with the Poisson equation

$$\frac{\partial E(z,t)}{\partial z} = \frac{e}{\epsilon}\left[n_+(z,t) - n_-(z,t) - n_e(z,t)\right],$$

and the boundary condition

$$\int_0^d E(z,t)\, dz = V \quad \forall t$$

Symbol	Units	Definition
n_+, n_-, n_e	m^{-3}	Positive, negative and electron densities, respectively
I	$m^{-3}\ s^{-1}$	Charge liberated per unit of time that escapes initial recombination
E	$V\ m^{-1}$	Electric field
γ	s^{-1}	Electron attachment
ϵ	$C\ V^{-1}\ m^{-1}$	Air permittivity

Table: Definition of the symbols used in the model equations.

[7] Paz-Martin, J. *Phys. Med.* **103**, 147-156 (2022).

Experimental setup

- Two PTW Advanced Markus (gap of 1 mm) and two PPC05 (gap of 0.6 mm) were irradiated in the 20 MeV electron beam at PTB MELAF facility.
- The dose per pulse (DPP) range from 41 mGy to 440 mGy.
- The reference dosimetry was performed using alanine pellets in combination with a flashDiamond prototype[8].
- The pulse duration was varied from 0.5 μs to 2.9 μs.

The charge collection efficiency (CCE) was obtained as follows:

$$\mathrm{CCE} = \frac{Q\, k_Q\, N_{D,w}^{^{60}\mathrm{Co}}}{D_w^Q}$$

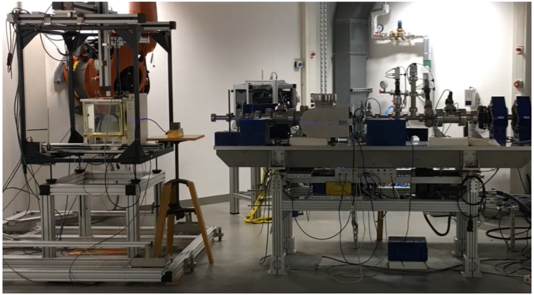

Figure: Experimental setup at PTB MELAF.

where Q is the collected charge corrected for pressure, temperature and polarity effect, k_Q is the beam quality correction factor obtained from Muir *et al.*[9] and $N_{D,w}^{^{60}\mathrm{Co}}$ is the absorbed dose to water calibration coefficient in ^{60}Co.

[7] Bourgouin, A. *et al. Phys. Med. Biol.* **67** 085013 (2022).
[8] Marinelli, M. *et al. Med. Phys* **49**(3) 1902-1910 (2022).
[9] Muir, B. R. and Rogers, D.W.O. *Med. Phys* **41**(11) 111701-7 (2014).

Results: Comparison between the models

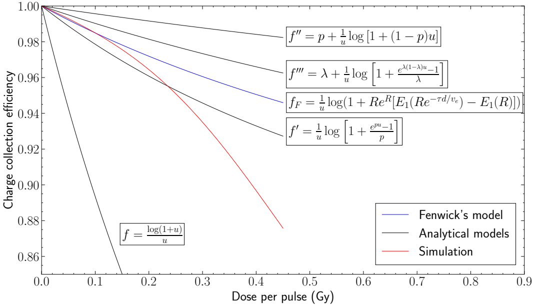

Figure: Comparison between the different Boag's models, the Fenwick model and the simulation for a 1 mm IC operated at 400 V

- When we use a instantaneous delivery the simulation and the Fenwick's model match each other in the very low dose per pulse regime.
- When the pulse duration is not instantaneous a small difference between simulation and Fenwick's model can be observed.

This numbers must be taken with care as this depends on operational voltage an IC gap.

Using a 1 mm ionization chamber operated at 400 V.

- The Fenwick's model shows to follow the simulation results better than the Boag's models, as expected.
- The Fenwick's model start to exhibit a different behavior when the dose per pulse becomes larger ($>$ 120 mGy for a 0.5 % difference) due to the electric field perturbation.

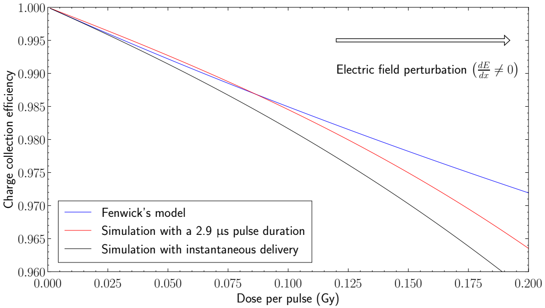

Figure: Comparison between the Fenwick model and the simulation for 2.9 µs pulse duration and a instantaneous delivery for a 1 mm IC operated at 400 V

Results: Comparison with experimental data from an Advanced Markus

- Experimental data shows much better agreement with simulation in terms of CCE: the contribution of free electrons is not negligible in this DPP range.
- The classical two voltage method shows to be clearly inaccurate predicting the k_{sat}.

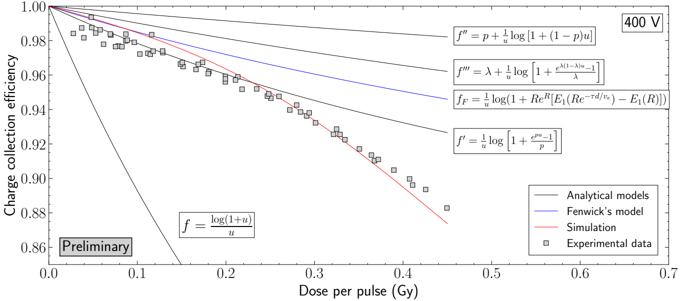

Figure: Comparison of the experimental charge collection efficiency against analytical models developed by Boag *et al.* and Fenwick for an Advanced Markus IC operated at 400 V.

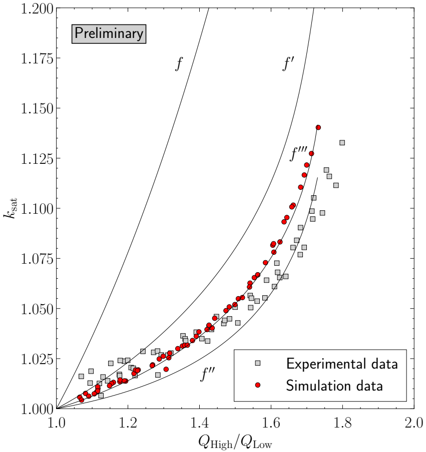

Figure: Two voltage method using the different Boag's formulas for a voltage quotient of 4 (400 V and 100 V).

Results: Comparison with experimental data from a PPC05

- As in the Advanced Markus chamber, the experimental data shows much better agreement with the simulation results, specially looking at the two voltage method.
- For this case, where the free electron fraction is higher the effect more noticeable ($p_{200\ V}^{\text{PPC05}} > p_{400\ V}^{\text{Adv.M.}}$)

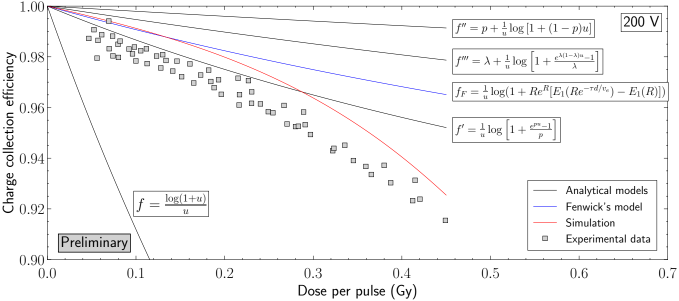

Figure: Comparison of the experimental charge collection efficiency against analytical models developed by Boag *et al.* and Fenwick for a PPC05 IC operated at 200 V.

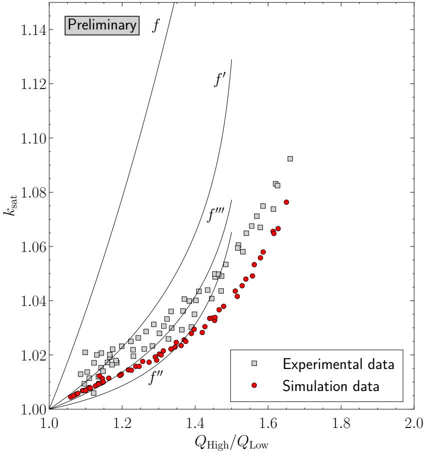

Figure: Two voltage method using the different Boag's formulas for a voltage quotient of 4 (200 V and 50 V).

Conclusions

- It has been shown that the different Boag's models present some inconsistencies even in the very low dose per pulse regime due to the modeling of the free electron distribution across the IC.
- The numerical models that are capable of describing the behavior of IC at ultra-high dose per pulse can be applied to very low dose per pulse.
- The exact analytical solution of for the recombination problem match the numerical solution in the low dose per pulse regime.
- The behavior of the two voltage method do not resemble the actual behavior of the IC due to the free electrons.
- A parametrization of the solution including free electrons similar to the current TRS-398 recommendations could be suitable for the clinical use.

The findings presented here encourage a revision of established methods for the evaluation of the saturation factor in conventional and new treatment modalities such as intra-operative radiotherapy.

Thank you very much for your attention! Do not hesitate to ask any question

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